The Phillips Curve and the Role of the Monetary Policy: A Cointegrated VAR Application to Chilean Data

Leonardo Salazar*

April 16, 2015

Abstract

In this paper the dynamics of inflation and unemployment are jointly analyzed as a system using the cointegrated vector of autoregression approach. The empirical analysis provides two main results. First, one cointegrating vector is interpreted as a Phillips curve augmented by productivity, that is, unemployment rate in excess of trend-adjusted productivity would lead to a downward pressure on inflation. Second, the equilibrium unemployment is time varying and its trajectory may be determined by the real interest rate and the level of productivity. This finding might confirm the persistence observed in unemployment and might be related to hysteresis found in previous studies in Chile. The fact that equilibrium unemployment may be affected by the interest rate seems to suggest that monetary policy is not completely neutral over the business cycle.

*leonardo.salazar@econ.ku.dk, Department of Economics, University of Copenhagen, Øster Farimagsgade 5, building 26, DK-1353, Copenhagen

1 Introduction

Since Phillips (1958) observed a negative relationship between wage inflation and unemployment rate, which became known as Phillips curve, numerous studies have empirically as well as theoretically analyzed this relationship. Over time, different formulations of the Phillips curve have appeared. These formulations analyze the relationship between inflation rate and some measure of the economic cycle. This analysis is mainly used to for the design of economic policies and forecasting inflation. A thorough review of this historical development of the Phillips curve can be found in Karanassou et al. (2010). These formulations can basically be divided in two groups: (a) standard Phillips curve models, and (b) New Keynesian Phillips curve models.

The first group, early in the sixties, studies the empirical regularity of a negative relationship between inflation rate and unemployment based on the traditional Phillips curve. This regularity was reported by Phillips (1958) for the UK and by Samuelson and Solow (1960) for the US. However, during the seventies this relationship broke down and a new formulation of the Phillips curve arose. Friedman (1968) and Phelps (1968) developed the expectations augmented Phillips curve, based on the idea that this curve shifts over time and in the long run the unemployment rate is independent of the rate of inflation, that is, there is a natural rate of unemployment acting as a long-run attractor for the unemployment rate. Therefore, under the expectations augmented Phillips curve, there is only short-run trade-off between inflation and unemployment rate. Furthermore, in these models the monetary policy has no effect on the equilibrium unemployment, both in the short and long run. This is known as the strong-form of the natural rate.

The second group relates actual and expected inflation to some measure of aggregate marginal cost instead of unemployment. Within this group one can distinguish between: (i) the standard New Keynesian Phillips curve model, and (ii) the frictional growth New Keynesian Phillips curve model. While the former model is consistent with the strong-form of the natural rate, the second one recognizes that monetary policy has only short-run effects on unemployment. This is known as the weak-form of the natural rate.

Regardless of the specification of the Phillips curve, most of the available studies share two empirical characteristics. First of all, the use of a single equation for modeling inflation, that is, normally the rate of inflation is assumed as the dependent variable explained by some indicator of the economic cycle (unemployment, production, marginal cost, etc.). A single equation approach can be justified by the classical dichotomy that nominal variables do not affect real variables, and that inflation and unemployment can be separately analyzed because there is not trade-off between them in the long run. However, the empirical evidence for this dichotomy seems quite weak; Fisher and Seater (1993), King and Watson (1994), Fair (2000), and Karanassou et al. (2005) present evidence of a significant long-run trade-off between inflation and unemployment rate. Furthermore, even if the classical dichotomy holds, the use of a single equation approach does not make use of feedback mechanisms embedded in the data.

Second, most empirical as well as theoretical studies assume the natural rate of unemployment as an exogenous variable, which could also be justified by the classical dichotomy. Normally, when a Phillips curve is estimated, the natural rate is assumed constant or variable; in the latter case, the natural rate is generally estimated by internally inconsistent procedures. For example, in a first stage the Phillips curve is estimated under the assumption of a constant natural rate, but then from the residuals of this equation a time-varying natural rate of unemployment is derived (Mankiw and Ball, 2002).

The results in this paper suggest that, when allowing for productivity, the dynamic of the inflation rate in Chile can be described by a Phillips curve. Furthermore, the natural rate of unemployment is time varying and exhibits a positive co-movement with the real interest rate, suggesting that the monetary policy might not be completely neutral over the business cycle. This finding is in line with the thought of Olivier Blanchard

> "...if we accept the fact that monetary policy can affect the real interest rate for a decade and perhaps more, then, we must accept, as a matter of logic, that it can affect activity, be it output or unemployment, for a roughly equal time" (Blanchard, 2003)

where implicitly is stated that monetary policy may have real effects on the economy.

Also, it is important to emphasize that most of Phillips curve studies are focused on industrialized economies, particularly on European countries, and only a small proportion in developing countries. Probably, the lack of studies in developing economies is due to scarceness of homogeneous databases covering long periods, and due the economic and politic instability in these economies. Whatever the reason, insufficiency of research in developing economies is harmful and hinders a suitable design of economic policies.

2 Literature review for Chile

There is scarcity of research about the Phillips curve in Chile. Normally, the available studies are used as a tool to evaluate transmission mechanism of monetary policy, inflation forecasting, and for the estimation of the natural rate of unemployment. Restrepo (2008) estimates different formulations of the Phillips curve to obtain the non-accelerating inflation rate of unemployment, NAIRU. The study covers the quarterly period 1987:03-1999:01 and uses as independent variables lags of the inflation rate, unemployment rate, supply shocks, and real exchange rate. Regardless of the Phillips curve specification, Restrepo finds a significant and negative relationship between inflation and cyclical unemployment, and between cyclical unemployment and inflation gap, which is interpreted as a short-run Phillips curve. Furthermore, based on a Granger causality test, this study suggests that causality goes from unemployment to inflation. In addition, the study emphasizes that the NAIRU is not constant, but its determinants are not analyzed.

Cabrera and Lagos (2000) estimate a Phillips curve to analyze the transmission mechanism of a monetary policy shock using term of trades, interest rate, output gap, nominal exchange rate, and core consumer price index as independent variables. Covering the monthly period 1986:02-1996:12 and based on an impulse-response exercise, the study concludes that the Phillips curve is not a proper tool to analyze the transmission mechanism of a monetary policy. This is because the GDP, among other variables, does not show a significant response to an increase in the monetary policy interest rate. Also, they show evidence of price puzzle. However, no further information is provided regarding the order of integration of the series, significance of the estimated coefficients, etc.

Estimations of the Phillips curve have also been used for forecasting inflation in Chile. De Simone (2001) estimates a Phillips curve with time-varying parameters where output gap is used as the independent variable. Covering the quarterly period 1990:01-1999:03, this study suggests that when the explicit inflation target, determined by the Central Bank of Chile, is used a proxy for inflation expectations, the estimated Phillips curve generates better forecasting of inflation than when this target is not included. The use of the output gap as a proxy for aggregate-demand effects instead of the unemployment gap is not justified in this study. This is also a common characteristic among numerous international studies. Unless a cointegrating relationship exists between output gap and unemployment gap, these variables should not be interchangeable in a Phillip curve.

Some studies have also analyzed the role of hysteresis to explain the persistence observed in the unemployment rate in Chile. Solimano and Larraín (2002) report evidence of hysteresis in unemployment using a single equation to estimate the unemployment rate dynamics. Covering the annual period 1960-2000 and using inflation rate, output gap, growth rate of productivity, and the lagged rate of unemployment as independent variables, the study concludes that hysteresis can explain the current unemployment. This result is based on the significance of the lagged unemployment rate in the regression.

A more recent study, by Gomes and da Silva (2008), tests the hypothesis of natural rate of employment versus the hysteresis hypothesis to explain the unemployment rate in Chile. Based on Lee and Strazicich (2003) two-break minimum LM unit root test, and using monthly observations 1982:02-2004:02, the study concludes that the null hypothesis of a unit root in the unemployment rate cannot be rejected. That is, the hysteresis hypothesis is better explaining the actual unemployment rate than the NAIRU. However, only a small part of the unemployment persistence can be explained by the hysteresis hypothesis.

Summarizing, the studies about the Phillips curve in Chile are mainly based on the estimation of a single equation where the dependent variable is inflation rate and as independent variables unemployment rate and other measures associated with the economic cycle are used. Generally, expected signs and significance are found in the estimated coefficients, but misspecification tests (normality, autocorrelation, etc.) are lacking. In addition, none of the studies report formal tests to analyze feedback effects between the variables. Some studies recognize the persistence of the unemployment rate over time and associate this with hysteresis.

This paper differs from the previous literature in the following sense (i) the dynamic of inflation and unemployment is jointly analyzed as a system. (ii) An econometric approach (cointegrated VAR) and theories (structural slumps and imperfect knowledge economics) consistent with the persistence observed in the data are used to support the main results, (iii) no prior restrictions are imposed in the information set, this allow the data to speak freely as possible about the underlying mechanism behind inflation and unemployment dynamics, and (iv) the variables explaining the persistence in unemployment rate are explicitly analyzed.

3 Theoretical framework

In this section the expectations-augmented Phillips curve is presented where supply shocks are allowed to shift the relationship between unemployment rate and inflation. This framework is developed in Hoover (2011) and assumes an economy with imperfect competition and imperfect information about the current price level.[1]

3.1 Price setting

A firm sets its price based on its expectation of the price level prevailing during the current period, and taking into account demand and supply conditions. That is, the price setting is written as

$$\triangle p_{j,t} = \triangle p_{j,t}^{e} + f\left(\text{demand factors}\right) + g\left(\text{supply factors}\right) \tag{1}$$

where $\triangle$ is the first difference operator, $p_{j,t} = \ln\left(P_{j,t}\right)$ and $P_{j,t}$ is the price set by firm j, $p_{j,t}^{e} = \ln\left(P_{j,t}^{e}\right)$ and $P_{j,t}^{e}$ is the expected level of price prevailing during the current period. This price is set by firm j at the end of period $t-1$ based on an information set available at the end of the same period, $Z_{j,t-1}$. The expected price can be written as $P_{j,t}^{e} = E_{j,t-1}\left[P_t | Z_{j,t-1}\right]$ where $E\left[\cdot\right]$ is the expectation operator. Functions $f\left(\cdot\right)$ and $g\left(\cdot\right)$ determine how demand and supply factors affect the pricing decision of firm j.

Equation (1) represents a single firm's price behavior. Taking the average of all firms in the economy, and assuming that demand and supply factors that are unique to particular firms average out, the economy's price behavior is written as

$$\triangle p_t = \triangle p_t^{e} + f\begin{pmatrix}\text{aggregate-demand} \\ \text{factors}\end{pmatrix} + g\begin{pmatrix}\text{aggregate-supply} \\ \text{factors}\end{pmatrix} \tag{2}$$

[1] In this section only the main results of the model are presented. For further details see chapter 15 in Hoover (2011).

where $\triangle p_t$ is the current inflation rate and $\triangle p_t^e$ is the average expectation of general price inflation for all firms. $f(\cdot)$ is reflecting demand-pull inflation and $g(\cdot)$ captures cost-push inflation.

3.2 The Phillips curve and the natural rate of unemployment

Functions $f(\cdot)$ and $g(\cdot)$ need to be explicitly defined in order to apply equation (2) to actual data. Since Phillips (1958), an accepted and usual measure of the aggregate demand has been the unemployment rate, that is

$$f\left(\text{aggregate-demand factors}\right) = a + bu_t \tag{3}$$

where u_t is the unemployment rate, b is assumed to be a negative constant given the countercyclical behavior of the unemployment rate, and $a > 0$.

Now, assuming for the moment that aggregate-supply factors can be ignored, an unemployment rate that equals the actual inflation to the expected inflation can be obtained by replacing (3) into (2), that is

$$u_t^\star = -\frac{a}{b} \tag{4}$$

where $u_t^\star$ is known as the natural rate of unemployment. If a and b are stable over time, the natural rate can be expressed without subscript t. Using the natural rate of unemployment, $u^\star$, equation (2) can be equivalently rewritten as

$$\triangle p_t = \triangle p_t^e - \gamma\left(u_t - u^\star\right) + g\left(\text{aggregate-supply factors}\right) \tag{5}$$

where $\gamma = -b$. Equation (5) is the Phillips curve extended to allow for supply shocks.

Equations (4) and (5) show two classical results: (i) the natural rate of unemployment is constant, and (ii) the Phillips curve is vertical at this level, that is, when expectations are fulfilled and the aggregate-supply factors are set at their "natural" levels, there is no long-run trade-off between unemployment and inflation.

3.3 Discussion

According to the theoretical framework, after a supply shock the unemployment rate should converge to its natural rate (4). This assumption is known as the natural rate hypothesis (NRH) and entails that the Phillips curve is vertical in the long run. However, the NRH does not seem to be empirically supported.[2]

Following an idea by Farmer (2013), the NRH can be tested in the following way: under the assumption that expectations are rational, the number of periods (for example quarters) where the actual inflation is above its expected value should be almost equal to the number of periods in where the opposite situation is observed. Then, over a decade, the average inflation rate should be almost equal to the average expected inflation. If the inflation rate over decades is plotted together with the unemployment rate, a vertical line at the natural rate of unemployment should be observed, supporting the NRH and rational expectations.

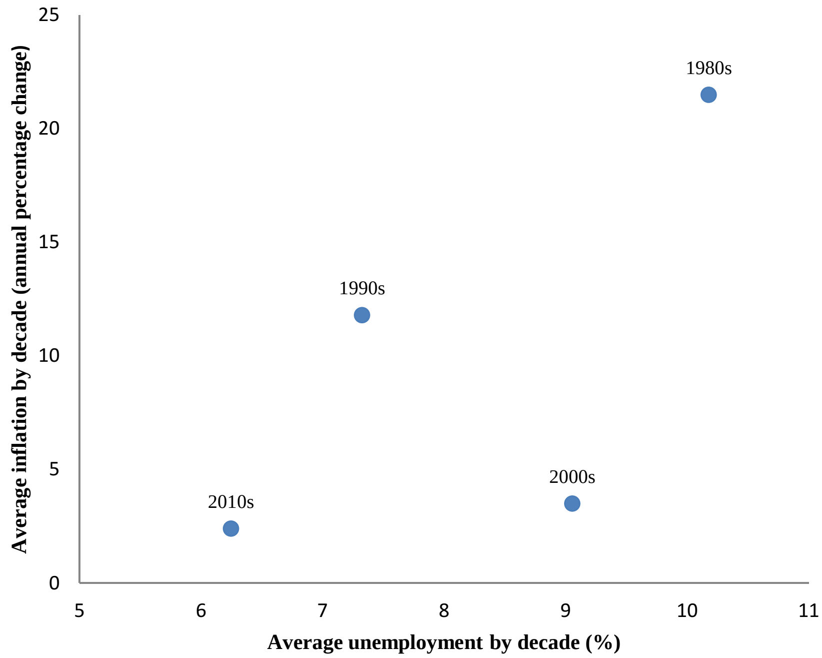
Figure 1 shows the average inflation and unemployment rate by decade for the Chilean economy.[3] The plotted points are not vertically aligned and there is no tendency for them to lie around a vertical line. Farmer (2013) obtains a similar result and categorically concludes that since expectations are unlikely to be systematically biased over decades, the NRH is false. However, a strong conclusion should not rely on a simple graph and further tests must be provided.

The result that unemployment does not converge to a unique constant value in the long run could potentially explain the persistence of this variable. Furthermore, this result may suggest the existence of a time-varying natural rate of unemployment. Phelps (1994), in his structural slumps theory, argues that the long swings observed in unemployment rates can be explained by fluctuations in exchange rates and real interest rates. Specifically, domestic real interest rates influence the natural rate of unemployment. That is, the natural rate of unemployment is time varying and its fluctuations reproduce the movements and persistence observed in real interest rates.

[2]See Farmer (2013) for the United States case, and Gomes and da Silva (2008) for the Chilean economy.

[3]1980s includes the period 1986-1989 and 2010s includes the period 2010-2013.

Figure 1: Average inflation and unemployment by decade in Chile*



*1980s includes 1986-1989, and 2010s includes 2010-2013.

Phelps provides two reasons for explaining the positive co-movement between the natural rate and inflation rate. First, higher real interest rates increase the natural rate of unemployment by discouraging investment, for example investment in the retention of workers (high interest rate reduces the probability of paying higher wages) or investment that could increase the productivity of the firm's workforce. Second, equilibrium employment will lessen with higher interest rates when government actions, given a real wage, reduce firms' labor demand, and by actions that affect the wealth of the working-age population, raising the real wage that workers demand (Aghion et al., 2003)

Phelps assumes a world where the unemployment rate and the interest rate are stationary. However, real interest rates are often found to be indistinguishable from a unit root process in empirical studies. Juselius and Juselius (2012) argue that the structural slumps theory based on imperfect knowledge economics[4] (IKE) expectations is more adequate to explain the persistent swings observed in the data.

Under IKE, while nominal interest rates exhibit strong persistence due to a non-stationary uncertainty premium, inflation rates are more stable over time, implying that the Fisher parity condition does not hold as a stationary condition. The uncertainty premium is generally related to the concept of "gap effect" which in the foreign currency market can be measured by the deviation of the real exchange rate from its long-run purchasing power parity value. In an IKE world, due to speculative behavior in the currency market, nominal exchange rates tend to move away from relative prices for long periods of time. That is, the real exchange rate behaves like a near $I(2)$ process. Therefore, the persistent deviations of the real exchange rate from its long-run benchmark value will be reflected in the uncertainty premium and hence in real interest rates.

An increase (decrease) in nominal interest rates will not be followed by an increase (decrease) in the consumer price inflation, generating a rise (drop) in the real interest rate. Therefore, the Fisher condition does not hold as a stationary condition. This is likely to result in massive inflows (outflows) of speculative capital, generating an appreciation (depreciation) of the real exchange rate and worsening (improving) the domestic competitiveness. Under this situation, domestic firms in the tradable sector cannot count on exchange rates to restore competitiveness after a shock to relative costs, e.g. a large wage rise. In this case, domestic firms will be prone to adjust profits rather than prices.

Profits can be adjusted through improvements in labor productivity by laying off the least productive part of the labor force. Thus, an increase in labor productivity and unemployment might be expected in periods of real appreciation and increasing real interest rates.[5]

Based on the above, a more adequate and general representation of the Phillips curve can be written as

$$\triangle p_t = \triangle p_t^e - \gamma\,(u_t - u_t^\star) + g\,(\text{aggregate-supply factors}) + \nu_t \qquad (6)$$

where ν_t is an stochastic error and the time-varying natural rate of unemployment can be expressed as a function of the real interest rate, ri. That

[4]See Frydman and Goldberg (2007) for further details.

[5]Further details about the Structural Slump theory and IKE are described in Juselius and Juselius (2012)

is, $u_t^\star = z\,(ri_t)$ and $z'\,(ri_t) > 0$.

4 Stylized facts

Panel (a) of Figure 2 shows the evolution of the unemployment rate, u, and inflation rate, $\triangle p$. The unemployment rate exhibits an important increase in 1998, possibly explained by the Asian Crisis[6] that seemed to hit the Chilean economy. After the Asian crisis the unemployment rate seems to exhibit a higher mean, suggesting that the mean of the natural rate of unemployment might have increased. The unemployment rate after the Asian crisis rose from an average rate of 6.9% to a rate of 8.4%. Another significant increase in the unemployment rate can be seen in 2009 when the financial crisis hit the Chilean economy. This increase seems transitory in contrast to the increase observed during the Asian crisis.[7]

Also in panel (a) of Figure 2, a clear gradual decrease in the inflation rate is observed over the sample, which might be associated with the implementation of inflation targeting in the middle of 1990. This policy allows the inflation rate to fluctuate in the range 2%-4%, centered in 3%, which has been more or less the case since 2000. The relation between unemployment rate and inflation is not easily discernible because the increase in unemployment rate in 1998 blurs the analysis. However, when controlling for this increase, the relationship is negative during most of the sample.

The unemployment rate is not the only variable that seems to be affected by the Asian crisis. In panel (b) of Figure 2, a deceleration in the real productivity, c, around 1998 can be observed. Productivity behaves like a trending variable and it seems that after 1998 there is a slowdown in the economic activity that might be associated with the Asian crisis. Unemployment rate and productivity exhibit seasonality caused by the agricultural activity in Chile which is higher during the last and first quarter of each year.

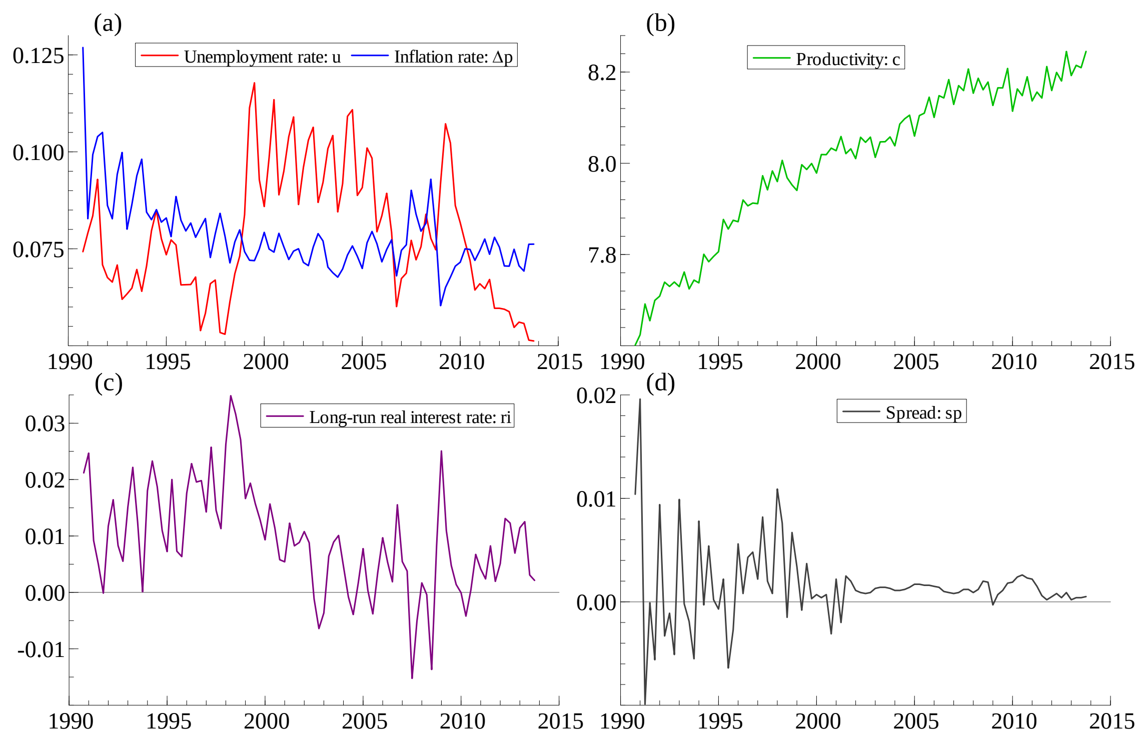
Between 2000 and 2001, several reforms were introduced in the financial market in Chile. In 2000, a law giving higher levels of protection to domestic

[6] The Asian crisis hit the Chilean economy in 1998. The tradable sector was the most affected since about 48% of the total exports were sent to Asia in 1998. The decrease in the Asian demand triggered the bankruptcy of many companies leading a large increase in the unemployment rate.

[7] After the Asian crisis, structural reforms were introduced in the labor market to reduce the impact of domestic and international shocks.

and foreign investors was promulgated. Also, in 2001 two laws were enacted, deregulating the financial system. In particular, the main deregulation was introduced in the capital account. The effects of the reforms are evident in panels (c) and (d) of Figure 2. Around 2001, a lower real interest rate, ri, and a decrease in the volatility of the interest rate spread, sp, between the long- and short-run interest rate can be observed. This may be related to the structural reforms introduced in the financial system in Chile.

Figure 2: Panel (a): unemployment rate and inflation rate. Panel (b): real productivity. Panel (c): long-run real interest rate. Panel (d): interest rate spread. Quarterly information 1990:4-2013:04



5 The empirical model analysis

5.1 Baseline model

The sample covers the quarterly period 1990:04-2013:04 and the following cointegrated VAR model is estimated for $\mathbf{x}_t' = [\triangle p_t, u_t, ri_t, sp_t, c_t]$

$$\triangle \mathbf{x}_t = \boldsymbol{\alpha}\tilde{\boldsymbol{\beta}}'\tilde{\mathbf{x}}_{t-1} + \boldsymbol{\Gamma}_1 \triangle \mathbf{x}_{t-1} + \sum_{i=0}^{1} \boldsymbol{\delta}_i d_{s98:03,t-i} + \boldsymbol{\delta}_2 \mathbf{D}_{p,t} + \boldsymbol{\delta}_3 \mathbf{S}_t + \boldsymbol{\varepsilon}_t \qquad (7)$$

where

- $\tilde{\mathbf{x}}_t = [\mathbf{x}_t, d_{s00:04,t}, t_1, t]'$, $\tilde{\boldsymbol{\beta}}' = [\boldsymbol{\beta}', \boldsymbol{\beta}_{01}, \boldsymbol{\beta}_{02}, \boldsymbol{\beta}_{03}]$
- $\triangle p_t$ is the inflation rate measured as $\triangle ln\,(\mathrm{CPI})_t$, where CPI is the consumer price index. Source: Central Bank of Chile.
- u_t is the unemployment rate measured as the ratio of unemployment to labor force. Source: Central Bank of Chile and National Statistics Institute of Chile.
- $ri_t = i_t^L - \triangle p_t$ is the long-run real interest rate. i_t^L is the long-run nominal interest rate. Source: Central Bank of Chile.
- $sp_t = i_t^L - i_t^S$ is the interest rate spread. i_t^S is the short-run nominal interest rate. Source: Central Bank of Chile. Following Juselius and Juselius (2012), the spread between the long- and short-run interest rate will be used as a proxy for expected inflation.
- c_t is the real labor productivity measured as the ratio of real GDP to the labor force. Source: Central Bank of Chile and National Statistics Institute of Chile.
- $d_{s00:04,t}$ is a step dummy restricted to be in the cointegrating relations. $d_{s00:04,t} = 1$ since 2000:04, 0 otherwise. This dummy accounts for the deregulation in financial markets in Chile (see panel (c) in Figure 2). The first difference of $d_{s00:04,t}$ is a blip dummy, taking the value 1 in 2000:04 and 0 in any other case. This blip dummy is an element in vector $\mathbf{D}_{p,t}$.

- t_1 is a broken linear trend restricted to the cointegration space, where $t_1 = 1, 2, \ldots, 62$ from 1998:03 until 2013:04, and 0 in any other case. The first difference of this broken linear trend is a step dummy, $d_{s98:03,t}$, taking the value 1 since 1998:03 and 0 in any other case. t_1 accounts for the productivity slowdown (see panel (b) in Figure 2).

- t is a deterministic trend restricted to be in the cointegrating relations. t accounts for the positive trend observed in productivity (see panel (b) in Figure 2).

- $\mathbf{D}_{p,t}$ and $\mathbf{S}_t$ are vectors of impulse dummies (0,0,0,1,0,0) and centered seasonal dummies, respectively.

- $\boldsymbol{\varepsilon}_t \overset{iid}{\sim} \mathcal{N}_{p=5}\left(\mathbf{0}, \boldsymbol{\Omega}\right)$

5.2 Misspecification tests and determination of the cointegration rank

Table 1 shows the residual misspecification tests of the baseline model (7).[8] The upper part shows that the model is, in general, well behaved. The hypotheses of non-autocorrelation and non-ARCH cannot be rejected; there are weak signs of non-normality since the null hypothesis can be rejected with a low p-value of 3%. The univariate tests, in the lower part of Table 1, suggest that only residual ARCH and signs of non-normality are presented on the interest rate spread. The ARCH problem is evident when looking at panel (d) of Figure 2. The normality problem in this equation is generated by excess of kurtosis rather than skewness. Despite these problems, the empirical analysis will be based on the specification of model (7) because for moderate excess of kurtosis, the VAR estimates are still robust (Gonzalo, 1994).

[8] Dennis (2006) provides a thorough description of the tests used in this paper and section.

Table 1: Misspecification tests CVAR model

Multivariate specification tests				
Autocorrelation		Normality	ARCH	
Order 1 : $\chi^2(25)$	Order 2 : $\chi^2(25)$	χ^2 (10)	Order 1 : $\chi^2(225)$	Order 2 : $\chi^2(450)$
16.18 [0.91]	26.59 [0.38]	19.85 [0.03]	215.69 [0.66]	498.57 [0.06]

Univariate specification tests					
	$\triangle^2 p_t$	$\triangle u_t$	$\triangle ri_t$	$\triangle sp_t$	$\triangle c_t$
ARCH Order 2:$\chi^2(2)$	0.55 [0.76]	7.04 [0.03]	1.28 [0.53]	13.06 [0.00]	0.62 [0.73]
Normality $\chi^2(2)$	2.22 [0.33]	1.19 [0.55]	0.18 [0.91]	8.64 [0.01]	1.03 [0.60]
Skewness	-0.31	0.27	0.00	0.15	-0.22
Kurtosis	3.33	2.92	2.94	4.26	3.08

[·] is the p-value of the test.

The upper part of Table 2 reports the Barlett corrected trace test, its corresponding p-value in brackets, and eigenvalues λ_i, for the null hypothesis of $r = 0, \ldots, 4$ cointegrating relations. The hypothesis $r = 4$ cannot be rejected based on a p-value of 20%. To check the adequacy of this choice, the lower part of Table 2 reports the four largest characteristics roots for the unrestricted model, $r = 5$, and for the restricted models based on $r = 1, \ldots, 4$. The unrestricted VAR has only one reasonably large root, 0.77, suggesting that the restricted model should not contain more than one unit root. When $r = 4$ this criterion is satisfied, leaving 0.67 as the largest root in the system. All the other models introduce additional persistence in the system, that is, generate more than one unit root. Therefore, based on the trace test and characteristic roots in the model, the following analysis is base on $r = 4$.

Table 2: Rank determination

$p-r$	$H_0 : r =$	Eigenvalues (λ_i)	Trace	p-value	$Q_{.95}$
		Trace test			
5	0	0.76	270.78	[0.00]	108.16
4	1	0.58	152.74	[0.00]	80.67
3	2	0.35	79.26	[0.00]	56.65
2	3	0.30	43.41	[0.01]	35.62
1	4	0.14	13.12	[0.20]	17.89
		Four largest characteristic roots			
4	1	1.00	1.00	1.00	1.00
3	2	1.00	1.00	1.00	0.51
2	3	1.00	1.00	0.55	0.55
1	4	1.00	0.67	0.67	0.51
0	5	0.77	0.67	0.67	0.52

$[\cdot]$ is the p-value of the Trace test simulated according to the baseline model (7).
$Q_{.95}$ is the 5% critical value of the Trace test.

5.3 Identification of the long-run structure

In order to identify the pulling forces, a set of restrictions must be imposed on $\tilde{\boldsymbol{\beta}}$. These restrictions can be represented through the hypothesis $\mathscr{H}_{\tilde{\boldsymbol{\beta}}} : \tilde{\boldsymbol{\beta}} = (\mathbf{H}_1\boldsymbol{\varphi}_1, \mathbf{H}_2\boldsymbol{\varphi}_2, \ldots, \mathbf{H}_r\boldsymbol{\varphi}_r)$ where $\mathbf{H}_i$ is a restriction matrix of dimension $(p_1 \times s_i)$, p_1 is the dimension of $\tilde{\mathbf{x}}$, and $p_1 - s_i$ is the number of restrictions imposed on $\tilde{\boldsymbol{\beta}}_i$. $\boldsymbol{\varphi}_i$ is a $(s_i \times 1)$ vector of unknown parameters and the test is asymptotically distributed as χ^2 with degrees of freedom equal to $\sum_{i=1}^{r} (p_1 - s_i - r + 1)$ (Johansen, 1996).

A set of restrictions imposed on $\tilde{\boldsymbol{\beta}}$ was not rejected based on $\chi^2\,(5) = 4.04$ with p-value of 54.3%. The over-identified structure on $\tilde{\boldsymbol{\beta}}$, together with the unrestricted estimates of $\boldsymbol{\alpha}$, is presented in Table 3. To facilitate the interpretation, an α_{ij} coefficient in bold face means that the cointegrating relation i is equilibrium correcting in the equation $\triangle\mathbf{x}_{i,t}$, $i = 1, \ldots, 5$ and $j = 1, \ldots, 4$, whereas an error increasing coefficient is given in italic.[9]

[9]When $\alpha_{ij}\beta_{ij} < 0$, the cointegrating relation is equilibrium correcting in the equation $\triangle\mathbf{x}_{i,t}$. Otherwise, the cointegrating relation describes an overshooting behavior in the equation $\triangle\mathbf{x}_{i,t}$ (Juselius, 2006). According to the results in Table 2, all characteristic roots are inside of the unit circle. Therefore, the system is stationary and any overshooting behavior is compensated by a correcting behavior.

Table 3: An over-identified structure on $\tilde{\boldsymbol{\beta}}$

	$\triangle p_t$	u_t	ri_t	sp_t	c_t	$d_{s00:04,t}$	t_1	t
$\tilde{\boldsymbol{\beta}}_1'$	1.00	0.22	-	-	−0.10	-	−1.99	2.57
		(5.54)			(−4.42)		(−11.14)	(9.98)
$\boldsymbol{\alpha}_1'$	**−0.85**	*0.45*	0.64	0.27	**1.61**			
	(−7.81)	(3.52)	(6.49)	(7.22)	(3.87)			
$\tilde{\boldsymbol{\beta}}_2'$	-	1.00	−4.18	-	0.20	−0.04	-	-
			(−21.45)		(9.76)	(−3.09)		
$\boldsymbol{\alpha}_2'$	$*$	**−0.39**	*−0.22*	−0.07	$*$			
		(−5.43)	(−3.89)	(−3.45)				
$\tilde{\boldsymbol{\beta}}_3'$	-	-	1.00	−1.00	-	0.01	-	-
						(3.73)		
$\boldsymbol{\alpha}_3'$	$*$	−1.62	**−1.44**	*−0.32*	$*$			
		(−4.13)	(−4.71)	(−2.84)				
$\tilde{\boldsymbol{\beta}}_4'$	0.19	-	-	1.00	−0.02	-	-	0.10
	(10.32)				(−5.45)			(4.99)
$\boldsymbol{\alpha}_4'$	*1.02*	−1.68	−2.04	**−1.42**	*−3.81*			
	(2.41)	(−3.38)	(−5.29)	(−9.83)	(−2.35)			

Note 1: $(\cdot)$ is the t-value. $*$ stands for an alpha coefficient with $|\text{t-value}| \leq 2.0$.

Note 2: "-" is a zero restriction.

Note 3: t_1 and t are scaled by a factor 10^{-3}.

The first cointegrating vector in Table 3, $\tilde{\boldsymbol{\beta}}_1'\tilde{\mathbf{x}}_t$, can be expressed as

$$\triangle p_t = -\underset{(5.54)}{0.22}\left(u_t - \tilde{c}_t\right) + \hat{\mu}_{1,t} \tag{8}$$

where $\tilde{c}_t = 0.45c_t + 9.04 \times 10^{-3}t_1 - 1.17 \times 10^{-2}t$ is the trend-adjusted productivity and $\hat{\mu}_{1,t} \sim I(0)$. Thus, equation (8) is a long-run relationship between inflation, unemployment rate, and trend-adjusted productivity. This result can be interpreted in the following way: when allowing for productivity, relation (8) describes a Phillips curve over the business cycle. That is, unemployment in excess of the trend-adjusted productivity, $\tilde{c}_t$, would lead a downward pressure on inflation rate. This relation is consistent with Juselius (2006), and Juselius and Ordóñez (2009) who find evidence of long-run co-movements between unemployment rate and trend-adjusted productivity.

The alpha coefficients related to equation (8), $\boldsymbol{\alpha}_1$, suggest that while inflation rate and productivity are equilibrium correcting to the Phillips curve, the unemployment rate is equilibrium error increasing. The latter is consistent with long and persistent swings in unemployment, possibly associated with long business cycles. The long-run interest rate and interest rate spread have positively reacted to relation (8).

The second relation in Table (3), $\tilde{\boldsymbol{\beta}}_2'\tilde{\mathbf{x}}_t$, can be written as

$$u_t = \underset{(21.45)}{4.18}\, ri_t - \underset{(9.76)}{0.20} c_t + \underset{(3.09)}{0.04} d_{s00:04,t} + \hat{\mu}_{2,t} \tag{9}$$

where $\hat{\mu}_{2,t} \sim I(0)$. This equation is a long-run relationship between unemployment rate, real interest rate, and the level of productivity. Equation (9) describes the unemployment rate that the economy reaches in the long run, that is, the natural rate (Mankiw and Ball, 2002).

There are several important features embedded in this relation. First, equation (9) shows that unemployment is not stationary *per se* and it needs to be combined with other variables to obtain a stationary relationship. This result confirms the persistence of the unemployment rate and might explain the hysteresis found in previous studies in Chile.

Second, equation (9) suggests that the natural rate of unemployment is not constant over the business cycle, that is, which is consistent with a time-varying natural rate. This finding is not in line with the constant natural rate predicted by equation (4) in the theoretical framework, but corroborates (i) the non-vertical scatter of the inflation rate and unemployment shown in Figure 1, and (ii) the general representation of the Phillips curve (6).

Third, when allowing for an equilibrium mean shift in interest rate in 2000:04, equation (9) shows a positive and significant co-movement between unemployment rate and real interest rate, corroborating the idea by Phelps (1994). That is, the equilibrium unemployment will increase with higher interest rates. Furthermore, given that the Central Bank of Chile conducts its monetary policy using the interest rate as the main tool to keep the inflation rate close to its target, equation (9) suggests that monetary policy in Chile might not be completely neutral over the business cycle.

Finally, equation (9) shows a negative co-movement between the unemployment rate and the level of productivity. This finding is in line with the empirical results in Ball and Moffitt (2001) and Staiger et al. (2001). This relationship has been studied using two approaches: the first assumes that there is a mismatch between the perception of productivity growth by firms and workers. While firms are assumed to directly observe the productivity growth trend, workers only infer this growth base on limited information. Then, an increase in productivity growth temporarily lowers inflation and the natural rate (Slacalek, 2004).

The second approach is associated with job search theories which suggest two opposite outcomes: (i) increases in productivity generate a higher value of a worker to the firm, stimulating job vacancies and reducing unemployment, and (ii) productivity growth may cause structural changes, destroying old jobs and replacing them by new ones. This mechanism reduces employment duration and increases the natural rate. Therefore, the final effect of productivity on the natural rate depends on the relative size of (i) and (ii).

The alpha coefficients, $\boldsymbol{\alpha}_2$, associated with the second cointegrating equation indicate that the interest rate spread has been negatively affected by relation (9) and that unemployment rate is equilibrium error correcting to this relation. The real interest rate is equilibrium error increasing to equation (9), which is consistent with the persistence observed in this variable.

The third relationship in Table 3, $\tilde{\boldsymbol{\beta}}_3' \tilde{\mathbf{x}}_t$, can be expressed as

$$ri_t = sp_t - \underset{(3.73)}{0.01} d_{s00:04,t} + \hat{\mu}_{3,t}$$

or, equivalently,

$$\left(i^S - \triangle p\right)_t = -\underset{(3.73)}{0.01} d_{s00:04,t} + \hat{\mu}_{3,t} \tag{10}$$

where $\hat{\mu}_{3,t} \sim I(0)$. Then, the third cointegrating relation describes a stationary short-run real interest rate when allowing for an equilibrium mean shift in 2000:04. This shift, measured by the step dummy $d_{s00:04,t}$, is reflecting the effect of the reforms introduced in the financial system in Chile. The alpha coefficients, $\boldsymbol{\alpha}_3$, suggest that unemployment rate has been negatively affected by equation (10). In addition, the real interest rate is equilibrium correcting to this equation, whereas the interest rate spread is error increasing.

Lastly, the fourth relation in Table 3, $\tilde{\boldsymbol{\beta}}_4' \tilde{\mathbf{x}}_t$, can be expressed as

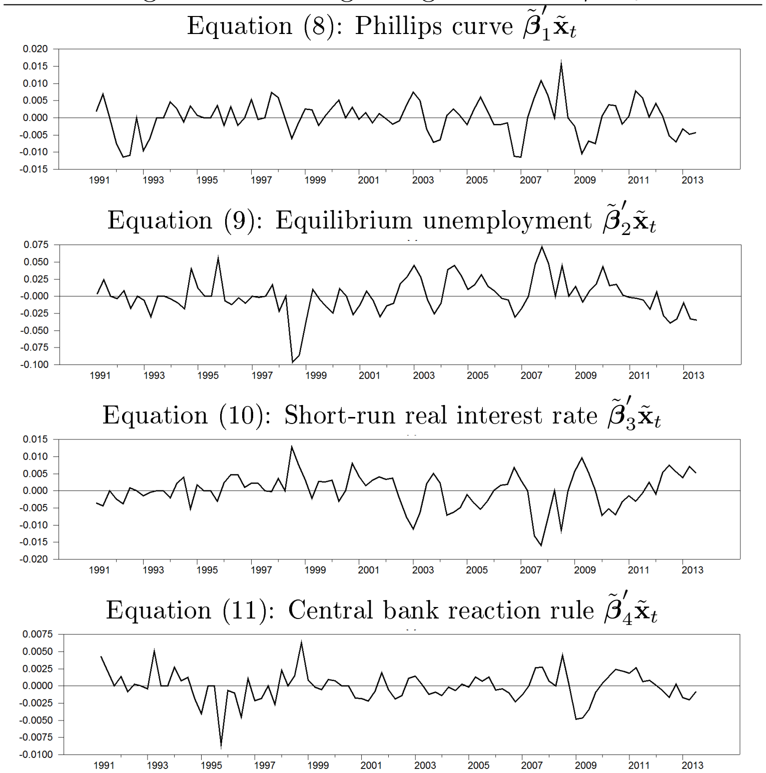
$$sp_t = -\underset{(10.32)}{0.19} \triangle p_t + \underset{(5.45)}{0.02} c_t - \underset{(4.99)}{0.10} \times 10^{-3} t + \hat{\mu}_{4,t} \tag{11}$$

where $\hat{\mu}_{4,t} \sim I(0)$. This is a long-run relationship between the interest rate spread, inflation, and trend-adjusted productivity. Given that the spread is equilibrium correcting to this relation, equation (11) can be interpreted as a central bank reaction rule. For example, the central bank may increase the short-run interest rate to counteract inflationary pressures due to excess demand associated with the business cycle. This is consistent with the countercyclical policy of the Central Bank in Chile. The alpha coefficients,

$\boldsymbol{\alpha}_4$, indicate that unemployment and real interest rate have been negatively affected by equation (11). Furthermore, the inflation rate and productivity are error equilibrium increasing to the reaction rule.

The cointegrating relations are shown in Figure 3.[10] This figure suggests that, despite some persistent deviations, all cointegrating relationships seem mean-reverting. Furthermore, Figure A.1 in the Appendix indicates that when $r = 4$ there is no signal of parameter non-constancy in model (7).

Figure 3: Cointegrating Relations $\tilde{\boldsymbol{\beta}}'\tilde{\mathbf{x}}_t$



[10] The graphs correspond to the cointegrating relationships in model (7) where the effects of the short-run dynamics, $\mathbf{\Gamma}_1 \triangle \mathbf{x}_{t-1}$, have been concentrated out. For further details see chapter 7 in Juselius (2006).

6 Policy implications

The empirical results in this paper suggest that monetary policy matters to equilibrium unemployment. According to the IKE theory's predictions, the long swings observed in nominal exchange rate around relative prices will be reflected in real interest rates. Furthermore, the structural slumps theory predicts that the domestic real interest rate influences the natural rate of unemployment. Therefore, the equilibrium unemployment may be affected by long lasting appreciation or depreciation in real exchange rates and/or by economic policies that have impact on interests rates, e.g. the monetary policy of central banks.

The main objective of the Central Bank of Chile is "safeguarding the stability of the currency and the normal functioning of the internal and external payment systems" (Section III, Ley Orgánica Constitucional del Banco Central de Chile, 1989). To achieve this objective, the Central Bank conducts its monetary policy based on inflation targeting complemented by floating exchange rate regime. The main instrument used to keep inflation close to its target is the monetary policy interest rate. The Central Bank is allowed to change this interest rate and these changes are passed to the interbank interest rate through open-markets operations, interest-bearing reserves, discount-window policy, etc. Finally, commercial banks pass these variations to lending and/or deposit rates which may change decisions about consumption, savings and investments. This affects aggregate demand and hence the price level in the economy.

It follows from the previous analysis that the main goal of the monetary policy in Chile is price stability. The central bank, empowered by law, can change the nominal interest rates to pursue its objective. That is, by increasing nominal interest rates, given an inflation rate, the central bank may shift aggregate demand, limiting price fluctuations. However, according to the empirical results in this paper, specifically equation (9), changes in real interest rate may have a significant effect on the steady-state unemployment.

Ball (2009) suggests that there is more than one level of unemployment compatible with a given inflation target. Then, a central bank might create unnecessary high unemployment in achieving its inflation target. Furthermore, Ball recommends that (i) during recessions central banks should ease their monetary policy and (ii) central banks facing high levels of unemployment should expand demand, accepting a rise in inflation to reduce the equilibrium unemployment.

The first of Ball's recommendations has been followed by the Central Bank of Chile since the partial implementation of the inflation targeting in 1990. The nature of this policy is countercyclical. That is, given that the economic cycle determines the short- and medium-term inflation, the monetary policy has a countercyclical influence in the inflation targeting system (Central Bank of Chile, 2007). Therefore, the monetary policy may reduce the volatility of inflation and output.

The second of Ball's recommendations seems problematic and it depends on the central bank's willingness to accept higher rates of inflation. If the cost of disinflation is larger than the benefits of a natural rate reduction, the second recommendation does not seem feasible. Furthermore, in the Chilean case, unemployment is not a central bank's target.

7 Conclusions

This paper has empirically analyzed the dynamic of inflation and unemployment in Chile using the cointegrated VAR approach. The results seem to suggest that one cointegrating vector can be interpreted as a Phillip curve. This curve describes a trade-off between inflation and unemployment when allowing for trend-adjusted productivity. That is, unemployment in excess of trend-adjusted productivity would lead to a downward pressure on inflation.

In addition, the empirical results suggest that equilibrium unemployment is time varying and its trajectory may be influenced by the real interest rate and productivity. This finding might be associated with the hysteresis found in previous studies in Chile. The fact that there is a positive co-movement between unemployment and interest rate may suggest that monetary policy is not completely neutral over the business cycle. This result is consistent with the structural slumps theory (Phelps, 1994), based on imperfect knowledge economics (IKE) expectations (Frydman and Goldberg, 2007).

The Central Bank of Chile conducts its monetary policy based on inflation targeting and the main instrument to keep the inflation rate close to its target is the interest rate. Given that the nature of the monetary policy in Chile is countercyclical, when the economy is growing over (under) its potential level, the Central Bank may increase (decrease) the interest rate to safeguard the stability of the currency. In doing so, the Central Bank might modify the trajectory of the equilibrium unemployment. That is, during economic expansions (contractions), an increase (decrease) in the natural rate of unemployment might occur.

References

(1989): *Ley Orgánica Constitucional del Banco Central de Chile. Ley No. 18.840.*

(2007): *Central Bank of Chile: Monetary Policy in an Inflation Targeting Framework.*

Aghion, P., R. Frydman, J. Stiglitz, and M. Woodford (2003): "Edmund S. Phelps and Modern Macroeconomics," *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, 3–22.

Ball, L. and R. Moffitt (2001): "Productivity growth and the Phillips curve," Tech. rep., National Bureau of Economic Research.

Ball, L. M. (2009): "Hysteresis in unemployment: old and new evidence," Tech. rep., National Bureau of Economic Research.

Blanchard, O. (2003): "Monetary policy and unemployment," in *Remarks at the Conference" Monetary Policy and the Labour Market: A Conference in Honor of James Tobin", New School University.*

Cabrera, A. and L. F. Lagos (2000): *Monetary Policy in Chile: A Black Box?*, vol. 88, Banco Central de Chile.

De Simone, F. N. (2001): "Proyección de la inflación en Chile," *Economía chilena*, 4, 59–85.

Dennis, J. G. (2006): *CATS in RATS Cointegration Analysis of Times Series. Version 2*, Estima.

Fair, R. C. (2000): "Testing the NAIRU model for the United States," *Review of Economics and Statistics*, 82, 64–71.

Farmer, R. E. (2013): "The Natural Rate Hypothesis: An idea past its sell-by date," Tech. rep., National Bureau of Economic Research.

Fisher, M. E. and J. J. Seater (1993): "Long-run neutrality and superneutrality in an ARIMA framework," *The American Economic Review*, 402–415.

Friedman, M. (1968): "The Role of Monetary Policy," *American Economic Review*, 58.

Frydman, R. and M. D. Goldberg (2007): *Imperfect knowledge economics: Exchange rates and risk*, Princeton University Press.

Gomes, F. and C. G. da Silva (2008): "Hysteresis vs. natural rate of unemployment in Brazil and Chile," *Applied Economics Letters*, 15, 53–56.

Gonzalo, J. (1994): "Five alternative methods of estimating long-run equilibrium relationships," *Journal of econometrics*, 60, 203–233.

Hoover, K. D. (2011): *Applied intermediate macroeconomics*, Cambridge University Press.

Johansen, S. (1996): *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*, Oxford University Press.

Juselius, K. (2006): *The cointegrated VAR model: methodology and applications*, Oxford University Press.

Juselius, K. and M. Juselius (2012): "Balance sheet recessions and time'varying coeffi cients in a Phillips curve relationship: An application to Finnish data," *Essays in Nonlinear Time series Econometrics.*

Juselius, K. and J. Ordóñez (2009): "Balassa-Samuelson and Wage, Price and Unemployment Dynamics in the Spanish Transition to EMU Membership," *Economics: The Open-Access, Open-Assessment E-Journal*, 3.

Karanassou, M., H. Sala, and D. J. Snower (2005): "A reappraisal of the inflation–unemployment tradeoff," *European Journal of Political Economy*, 21, 1–32.

——— (2010): "Phillips curves and unemployment dynamics: a critique and a holistic perspective," *Journal of Economic Surveys*, 24, 1–51.

King, R. G. and M. W. Watson (1994): "The post-war US Phillips curve: a revisionist econometric history," in *Carnegie-Rochester Conference Series on Public Policy*, Elsevier, vol. 41, 157–219.

Lee, J. and M. C. Strazicich (2003): "Minimum Lagrange multiplier unit root test with two structural breaks," *Review of Economics and Statistics*, 85, 1082–1089.

Mankiw, N. G. and L. Ball (2002): "The NAIRU in theory and practice," *Journal of Economic Perspectives*, 16.

Phelps, E. S. (1968): "Money-wage dynamics and labor-market equilibrium," *The Journal of Political Economy*, 678–711.

——— (1994): *Structural slumps: The modern equilibrium theory of unemployment, interest, and assets*, Harvard University Press.

Phillips, A. W. (1958): "The Relation Between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–19571," *economica*, 25, 283–299.

Restrepo, J. (2008): "Estimaciones de la NAIRU para Chile," *Economía chilena*, 11, 31–46.

Samuelson, P. A. and R. M. Solow (1960): "Analytical aspects of anti-inflation policy," *The American Economic Review*, 177–194.

Slacalek, J. (2004): "Productivity and the Natural Rate of Unemployment," Tech. rep., DIW-Diskussionspapiere.

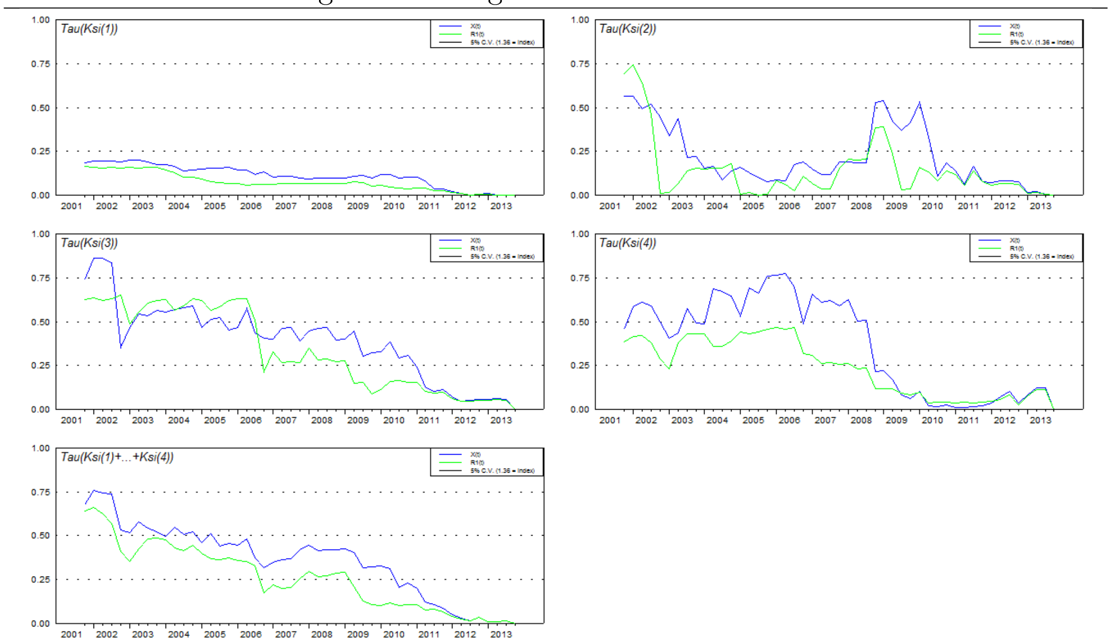
Solimano, A. and G. Larraín (2002): "From Economic Miracle to Sluggish Performance: Employment, Unemployment and Growth in the Chilean Economy," *ILO multidisciplinary Team Santiago de Chile. Santiago de Chile, International Labour Office, mimeo.*

Staiger, D., J. H. Stock, and M. W. Watson (2001): "Prices, Wages and the US NAIRU in the 1990s," Tech. rep., National bureau of economic research.

Appendix: Fluctuation test

Figure A.1 shows the eigenvalue fluctuation test for each individual eigenvalue λ_i, $i = 1, 2, 3, 4$, and for the weighted average of them. The individual fluctuation tests correspond to $\text{Tau}\,(\text{Ksi}\,(i))$ and the weighted average to $\text{Tau}\,(\text{Ksi}\,(1) + \cdots + \text{Ksi}\,(4))$.[11] When the graph is above the unit line, the parameter constancy can be rejected at the 5% level. Based on this critical value, Figure A.1 suggests that there are no signs of parameter-non constancy in the model. This is valid for the full model (7), corresponding to the $X\,(t)$ graph, and for the concentrated model, represented by $R1\,(t)$ the graph where the short-run effects, $\mathbf{\Gamma}_1 \triangle \mathbf{x}_t$, have been concentrated out of the full model.

Figure A.1: Eigenvalue fluctuation test



[11] For further details about the eigenvalue fluctuation test, see chapter 9 in Juselius (2006)