'Love of Wealth' and Economic Growth

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Abstract

In a Ramsey-Cass-Koopmans growth framework it is shown that for an optimum the social planner cannot have an excessive *love of wealth*. With a 'right' *love of wealth* an optimum exists and implies higher long-run per capita capital, income and consumption relative to the standard model. This has important implication for comparative development trajectories. The optimum implies dynamic efficiency with the possibility to get arbitrarily close to the Golden Rule where long-run per capita consumption is maximal. It is shown that the optimal path is attaining its steady state more slowly. Thus, the beneficial effects of *love of wealth* materialize later than in the standard model. Furthermore, the economy can be decentralized as a competitive private ownership economy. One can then identify *love of wealth* with the "spirit of capitalism". The paper, hence, implies that one needs a 'right' level of the "spirit of capitalism" to realize any beneficial effects for the long run.

KEYWORDS: Economic Growth, Love of Wealth, Preference Shifts

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1 Introduction

It is generally recognized that people derive satisfaction from the holding of wealth. For instance, Weber (1930) and Pigou (1941) argue that individuals derive utility from the mere possession of wealth and not simply its expenditure.[1] Clearly, putting wealth directly into the welfare (utility) function may allow to capture important and quite realistic aspects of investment behaviour.[2]

Of course, investment behaviour is related to economic growth. In a seminal article Kurz (1968) has analyzed an optimal growth model in the spirit of Ramsey (1928), Cass (1965) and Koopmans (1965) where agents' utility includes utility from holding wealth. Keeping the analysis quite general Kurz shows that many different and complicated equilibria and transition dynamics may occur. Unfortunately, his paper does not present much economic intuition for his mathematical findings.

Simplified versions of his analysis were subsequently complemented by more economic interpretations. For example, Zou (1994), Bakshi and Chen (1996) and Carroll (2000) relate to Max Weber and argue that the dependence of utility on wealth captures the "*spirit of capitalism*". Zou uses absolute, and Bakshi and Chen relative wealth in their setups. In this paper I primarily relate to Zou's optimal growth framework.

For the latter it is well known that the long-run equilibrium obeys the *Modified Golden Rule*, under which the steady state capital stock is lower than the one associated with the *Golden Rule*, derived by Phelps (1961), which would yield maximum long-run, per capita consumption. Zou (1994) shows that utility derived from consumption and capital, where the latter is called *love of wealth* (LOW) in this paper, leads to an optimal path with a higher, steady state capital stock than under the Modified Golden Rule.[3] Corneo and Jeanne (1997) and Corneo and Jeanne (2001) show in an *endogenous growth* framework that love of (relative) wealth ("social status concerns" in their paper) can lead to an accumulation path that may even obey the Golden Rule. They also show that it is possible that too much love of wealth would entail over-

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[1]For example, an early contribution that introduced wealth in the utility function in order to model investment behaviour is Markowitz (1952). For recent papers that include love of wealth in their models see, for example, Kaplow (2009) and Kumhof and Rancière (2010).

[2]One question is whether it is the relative wealth or the absolute wealth that enters utility. The former allows to concentrate on status concerns, while the latter focuses on a form of 'pure' love of wealth. Both approaches can be found in the literature and I will relate to them below.

[3]The expression *love of wealth*, of course, draws on Plutarch's (46 AD-120 AD) essay "Περὶ φιλοπλουτίας" ("De Cupiditate Divitiarum" or "On the Love of Wealth") in his *Moralia* that the interested reader may wish to have a look at.

accumulation and would be socially undesirable. This is usually called "dynamically inefficient".

Against this background the present paper derives complementary results. In order to obtain those the paper concentrates on a simple logarithmic utility, Cobb-Douglas technology economy with a benevolent social planner that derives utility from per capita wealth and consumption. Thus, the model builds on Ramsey (1928), Cass (1965) and Koopmans (1965) who also analyze the problem of a central planning authority (social planner).[4] This makes it possible to get clean analytic expressions that serve to capture the essence of what love of wealth in a Solow (1956) growth context might imply. The analysis concentrates on steady states, but additionally on transitional dynamics which some earlier contributions have often not focussed on.[5]

In this context, the model yields the following results. There is no optimal accumulation path when there is excessive love of wealth. As the model's social planner is taken to care about both per capita consumption and wealth, an excessive zeal for accumulation would entail reductions in long-run consumption. Intuitively, the crave for building ever more pyramids would have to be met with lower consumption in the long run. As intuition suggests this is not a long-run optimum.

There exists a critical level of the love of wealth (LOW) below which optimal paths exist. These paths are such that in a steady state per capita wealth and consumption would be higher than in an economy where there is no LOW. Thus, somewhat surprisingly, having the 'right' craving for capital also leads to higher long-run consumption. In that sense love of wealth is quite beneficial. It implies that a simple preference shift from non-LOW to LOW allows for a better material situation, even for the consumers. However, in relative terms this is not so, because, even though the cake will be bigger, the steady state consumption share will be lower, the investment share higher and the return to capital lower in a LOW economy in comparison to the standard, optimal growth (non-LOW) economy.[6]

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[4]Later in the paper we will look at the implication of the paper's results for decentralized economies. The advantage to work with a social planner is that it may capture more general economic arrangements, of which a competitive decentralization is just one possibility.

[5]For instance, Wirl (1994) concentrates on transitional dynamics and the possibility of growth cycles when wealth enters utility. However, his paper concentrates rather on stability properties of such economies. In the present paper the focus is more on steady state properties and the speed of convergence in relation to standard optimal growth models.

[6]Empirically this implies that economies may exhibit quite different long-run income levels in a cross section. According to the model this would be entirely due to preference and not technology differences.

In this model the LOW which implies optimal paths is always accompanied by "dynamic efficiency". In fact, (arbitrarily) close to the critical level of the love of wealth, the capital stock implied by the Golden Rule and that chosen by a social planner with a LOW near that critical level would be almost identical. The reason is that in a standard optimal growth (non-LOW) model, impatience (the rate of time preference) is such that a steady state capital stock is chosen that is less than the one that maximizes steady state consumption. Love of wealth provides a counteracting effect. Thus, it is possible, even in a neoclassical, optimal growth context, to get arbitrarily close to the Golden Rule. Hence, the model implies some form of tradeoff between impatience and love of wealth. Less patience can be compensated by more love of wealth if one wants to have the same level of steady state capital.

Next, by log-linearizing the economy with LOW preferences around its steady state it is shown that the economy features saddle path stability. This is not too surprising, given the model's structure. However, an interesting finding here is that convergence in an economy with LOW preferences occurs more slowly than in the standard non-LOW case. Thus, two economies with these differences in preferences would converge differently to their respective steady states. The economy with LOW preferences would take longer, but have a higher capital stock in the long run.[7]

This allows one to argue that transitions of economies can be seen as having been started off by preference shifts. Suppose an economy without LOW preferences were in its steady state. Now assume that the social planner suddenly starts loving wealth and continues to do so forever. In a qualitative analysis using a phase diagram it turns out that taking the former steady state capital stock as the initial capital stock for getting into the new equilibrium reveals the following quite intuitive results. The economy will jump onto its new saddle path with lower consumption and higher investment, compared to the previous regime with no LOW. So on impact consumers will suffer as more investment is needed to obtain the new and higher steady state capital stock associated with LOW preferences. There will be one point in time where the consumers will be just as well off as before the preference shift. After that date consumers will unambiguously benefit from the preference shift.

These results are interesting because they may contribute to explanations what may have happened in transition economies. Examples that may come to mind are the Eastern European countries, Russia, China, India etc. For example, the model may

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[7]To my knowledge of the literature this appears to be a novel finding. Of course, it holds only under the assumptions made.

contribute to explanations why China, India and other economies have had such high investment rates over a longer period.

The logic of preference shifts towards capital might also apply to historical contexts. Here the claim is that preference shifts, similar to those analyzed here, were at work in the take-offs shortly before and during the Industrial Revolution. Of course, the model is too coarse to capture that in detail, but it may serve to highlight the possibility of contemplating preference shifts as (perhaps additional) initial movers of economic transitions.

Next, the paper invokes the second welfare theorem and argues that the contemplated economy can be decentralized as a competitive general equilibrium of a private ownership economy. See, for example, Acemoglu (2009), ch. 5. Thus, the social planner solution can quite straightforwardly be turned into the outcome of a market economy, given the assumptions about preferences and technology. This would require that the agents would have to have the same preferences as the social planner contemplated in this paper and where the social planner would have represented the agents' welfare in a benevolent way. Thus, under these circumstances all the results under the planner's solution would also hold for the decentralized economy.

For the decentralized economy one may reasonably interpret the more general term "love of wealth" as representing the "spirit of capitalism" as in, for example, Zou (1994).[8] Thus, for a competitive market economy we may then conclude that the 'right' level of the "spirit of capitalism" is good in terms of long-run income and consumption. But it may take a little longer to realize these effects in comparison to an economy with less of that spirit. Furthermore, distinct ('right') "spirits of capitalism" may yield quite different economic outcomes in the long-run.[9]

However, one of the paper's main insights is that an *excessive* "spirit of capitalism" is not optimal in that it cannot really be realized as a long-run optimum in a dynamic economy.

The paper is organized as follows. Sections 2 and 3 present the model and derive the social planner's optimal decision rules. Sections 4 and 5 analyze the steady state, and the transitional dynamics. Section 6 discusses decentralization, and section 7 concludes.

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[8]Notice that Zou contemplates a decentralized economy and sometimes uses a single, constant parameter, as in the present paper, to represent the "spirit of capitalism".

[9]That there are different forms of capitalism is, of course, well known. See, for example, Hall and Soskice (eds.) (2001).

2 The Model

The economy consists of a benevolent social planner, and many infinitely-lived individuals. The population is normalized so that we can think of the economy as consisting or a representative consumer. The model abstracts from uncertainty, technological progress, and population growth. By assumption the agents supply one unit of unskilled labour inelastically.

The social planner is taken to care about the representative individual, and faces the following resource constraint

$$\frac{k(t)}{dt} = y(t) - c(t) - \delta k(t) \tag{1}$$

where $k(t)$ denotes the (per capita) capital stock, $y(t)$ is per capita income and $c(t)$ per capita consumption. Furthermore, δ represents the constant depreciation rate of physical capital.[10]

The social planner has access to a technology which features constant returns to scale where aggregate output $Y(t)$ is produced using capital and labour, i.e. $Y(t) = F(K(t), L(t))$ where $K(t)$ and $L(t)$ denote the aggregate capital stock and labour input, respectively. For simplicity we consider a Cobb-Douglas economy with

$$Y(t) = K(t)^\alpha L(t)^{1-\alpha}, \quad \text{where } 0 < \alpha < 1.$$

Then we know that per capita output $y(t) \equiv Y(t)/(L(t)$ is given by

$$y(t) = f(k(t)) = k(t)^\alpha \tag{2}$$

where $k(t) \equiv K(t)/L(t)$ and we assume that there is full employment so that the population equals the number of persons working.

The social planner rewards factors according to their marginal products. Denoting the reward to capital by $r(t)$ and that for labour by $w(t)$ one gets[11]

$$r(t) = \alpha k(t)^{\alpha-1} \quad \text{and} \quad w(t) = f(k(t)) - r(t)k(t) = (1-\alpha)k(t)^\alpha \tag{3}$$

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[10]We denote $\frac{dk(t)}{dt}$ by $\dot{k}(t)$. Partial derivatives will be denoted by subscripts, for instance, for a function $f(x)$ we let $f_x \equiv \frac{\partial f}{\partial x}$, all evaluated at time t.

[11]Under competitive conditions und profit maximization the rates of return for capital and the wage rate would also equal the marginal products.

where the latter equality follows from the assumption of constant returns to scale.

From now on I will drop the indication that variables depend on time for convenience. When there may be cases where it needs to be made explicit, I will do so.

3 A Wealth-Loving Social Planner

Consider a social planner that maximizes an intertemporal utility stream subject to the economy's resource constraint in (1). The welfare stream for the social planner is given by

$$\int_0^\infty u(c,k) \, e^{-\rho t} dt$$

where we assume the following for the period utility function $u(\cdot)$

$$u_c > 0, \qquad u_{cc} < 0, \qquad u_k > 0 \qquad \text{and} \quad u_{kk} < 0.$$
$$\lim_{c\to 0} u_c = \infty, \quad \lim_{c\to\infty} u_c = 0, \quad \lim_{k\to 0} u_k = \infty \quad \text{and} \lim_{k\to\infty} u_k = 0.$$

Thus, period utility depends in a conventional way on (per capita) consumption c. What is different from usual problems is the fact that we assume that the social planner derives welfare from (per capita) wealth (capital).[12] Thus, $u(\cdot)$ is a function of k. This captures the fact that many people attach some value to wealth and capital per se. For instance, many people like to look at and visit impressive buildings (e.g. the Eiffel Tower, or the Empire State Building etc.) and derive utility from that. Also, many firms offer guided tours through their often very impressive plants of production such as e.g. the Boeing assembly halls in Seattle or Volkswagen's "Auto Manufaktur" in Dresden.[13]

The assumption that welfare is increasing in wealth is perhaps more problematic. Clearly, there are cases where additional capital may be valued less. An example may be the construction of an additional nuclear power plant. However, k is an index of all

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[12]Notice there is a literature that makes preferences dependent on relative wealth. There is basic idea is that optimizers are comparing themselves with others. Here that most interesting question is not taken up. Instead we focus on a planner that only looks at his or her own wealth. But this allows for comparisons, too, because below we will conduct comparative static exercises by which one contrasts planners with a higher and a lower preference for wealth.

[13]Clearly, marvelling at building from outside means that these buildings or plants have a public good nature. However, visiting them usually requires a fee to be paid so that buildings and guided tours then have a private good nature.

sorts of capital stocks. Most evidence would suggest that people generally like wealth and especially more of it. Otherwise, they would not do the things one can observe to increase their wealth. Of course, this is a perennial phenomenon. Thus, drawing on this 'stylized fact' may justify the assumption that welfare is increasing in wealth, $u_k > 0$.

The next assumption states that the marginal welfare of wealth is decreasing. Thus, the welfare gain becomes smaller as wealth increases. This may capture the observation that very rich people often say that an additional "palazzo" may not make them much happier, especially in comparison to the first one they already call their own.

The Inada conditions on welfare's reaction on the effects of wealth when there is hardly any or too much capital are not really necessary for most of the analysis below, but can be rationalized on quite intuitive grounds. For example, $\lim_{k \to 0} u_k = \infty$ would say that one is really craving for wealth if one does not have any. Here the claim is that many people would support that view. In turn, $\lim_{k \to \infty} u_k = 0$ would imply that Bill Gates does not really care if he gets an additional computer.

With all these assumptions the social planner's problem really becomes

$$\max_c \int_0^\infty u(c,k)\, e^{-\rho t} dt$$
$$s.t. \quad \dot{k} = f(k) - c - \delta k; \quad k_0 > 0 \ \text{and given.}$$

A general problem of this kind is studied by Kurz (1968). He shows that many different trajectories may or may not lead to balanced growth paths. The potential solutions of this problem depend on the exact form of the period utility function $u(c, k)$, and, in particular, on the assumed substitution of consumption and capital in period utility.

In this paper one of the main questions that is being analyzed is how the steady state and the speed of convergence are affected when moving from a regime where the social planner does not love capital to one where he or she does. To analyze that question I will restrict the analysis to the log-utility case under the assumption that the period utility derived from capital and consumption are separable. Thus, it is assumed that the social planner values each marginal increment of one component of utility essentially independently of the increment of the other component. Thus, consider the following period utility function

$$u(c,k) = \ln c + \gamma \ln k \quad \text{where} \quad \gamma \in [0, \infty). \tag{4}$$

One easily verifies that this period utility function satisfies all the properties that the more general setup requires. Here γ measures the preference of the social planner for wealth (capital) in comparison to that for consumption. If $\gamma = 0$, the social planner attaches no direct value to capital. If $\gamma \to \infty$, the social planner is only concerned about capital.

Thus, under the assumption of the period utility function in equation (4) the social planner's problem is given by

$$
\max_{c} \int_0^\infty \left[\ln c + \gamma \ln k\right] e^{-\rho t} dt \tag{5}
$$
$$
s.t. \quad \dot{k} = f(k) - c - \delta k, \quad k_0 = \text{given}.
$$

where ρ represents the constant rate of time preference.

The solution to this problem is obtained from the current value Hamiltonian

$$
H = \left[\ln c + \gamma \ln k\right] + \lambda \left[f(k) - c - \delta k\right]
$$

and the first order necessary conditions for its maximization entail the following equations have to be fulfilled

$$
u_c = \lambda \tag{6}
$$
$$
\dot{\lambda} = \lambda \rho - u_k - \lambda(f' - \delta) \tag{7}
$$

plus the requirement that the resource constraint in (1) and the transversality condition $\lim_{t \to \infty} k\lambda e^{-\rho t} = 0$ be satisfied.[14]

Under the assumptions made equations (6) and (7) can then be rearranged to find the social planner's optimal consumption growth rate

$$
\frac{\dot{c}}{c} = f'(k) + u_k/u_c - (\rho + \delta) = f'(k) + \gamma \cdot \frac{c}{k} - (\rho + \delta). \tag{8}
$$

Notice that, as is standard, consumption growth depends on the return to capital and the rate of time preference. Additionally, in this model the marginal rate of substitution (in welfare) between consumption and wealth matters too. In our log-utility case this is given by $-\frac{dc}{dk} = \frac{u_k}{u_c} = \gamma \cdot \frac{c}{k}$ and has a positive bearing on the (optimal) consumption

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[14]One easily verifies that the Hamiltonian is concave in c and k so that by Mangasarian's Theorem the sufficient conditions for a maximum would also be be satisfied.

growth rate. Furthermore, we note that, if there is positive consumption growth, the marginal product will be lower than in a conventional optimal growth model, where it would equal the sum of the depreciation and the time preference rate.

3.1 The transversality condition

The optimal consumption plan requires that the value of capital be zero at the end of the planning horizon. That imposes the requirement that

$$\lim_{t \to \infty} k_t \cdot \lambda_t \cdot e^{-\rho t} = 0 \tag{9}$$

hold. From equation (7) we know

$$\dot{\lambda} = \lambda(-f' + (\rho + \delta)) - \frac{\gamma}{k}.$$

Integrating this expression from time 0 up to time t yields

$$\lambda_t = e^{\int_0^t (\rho - f'(k_v) + \delta) dv} \left(\lambda_0 - \int_0^t \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds \right).$$

Inserting this into the transversality condition in (9) yields

$$\lim_{t \to \infty} k_t \cdot e^{\int_0^t (\rho - f'(k_v) + \delta) dv} \left(\lambda_0 - \int_0^t \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds \right) \cdot e^{-\rho t} = 0.$$

as the condition to be satisfied. Noting that $e^{\int_0^t \rho dv} = e^{\rho t}$ we get

$$\lim_{t \to \infty} k_t \cdot e^{-\int_0^t (f'(k_v) - \delta) dv} \cdot \left(\lambda_0 - \int_0^t \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds \right) = 0.$$

Clearly, this condition is equivalent to

$$\lim_{t \to \infty} k_t \cdot \lim_{t \to \infty} e^{-\int_0^t (f'(k_v) - \delta) dv} \cdot \left(\lambda_0 - \lim_{t \to \infty} \int_0^t \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds \right) = 0.$$

Jumping ahead it will turn out that in a long-run equilibrium $\lim_{t \to \infty} k_t = k^* < \infty$, that is, the steady state capital stock k^* will be finite. Then the product of the other limits must vanish asymptotically. With a finite capital stock in the long-run equilibrium the second limit requires $\lim_{t \to \infty} e^{-\int_0^t (f'(k_v) - \delta) dv} = 0$ so that $f'(k_\infty) > \delta$ in equilibrium is

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needed and $k_\infty = k^*$. See appendix A for more details. λ_0 is a finite and positive number. Thus, the product of λ_0, the first and the second limit will be zero asymptotically, if $f'(k^*) > \delta$.

As regards the third limit it is shown in appendix A that the integral

$$\int_0^\infty \left(\frac{\gamma}{k_s}\right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds$$

converges to a finite number. Thus, the product of this and the other two limits will be zero asymptotically, if $f'(k^*) > \delta$, so that the transversality condition is indeed met.

Lemma 1 *The transversality condition requires that $f'(k^*) > \delta$ where k^* denotes the steady state capital stock.*

As will be shown in more detail below, this is true in the steady state. The lemma has the important implication that optimality requires that the social planner choose a dynamically efficient solution.

4 The steady state

In the long-run, steady state equilibrium with balanced growth the economy is characterized by the condition $\dot{k}/k = \dot{c}/c = 0$. Using (1) and (8) for $\dot{k}/k = \dot{c}/c$ requires that

$$
\begin{aligned}
f'(k) - (\rho + \delta) + \gamma \frac{c}{k} &= \frac{f(k)}{k} - \frac{c}{k} - \delta \\
(\gamma + 1)c &= f(k) - f'(k) \cdot k + \rho k \\
c^* &= (w + \rho k) \left(\frac{1}{\gamma + 1}\right)
\end{aligned}
\tag{10}
$$

holds. Thus, consumption in steady state, c^*, depends on wage income and capital income net of investment outlays and on the welfare weight γ, which captures the preference for capital. For given w, ρ and k, steady state consumption c^* is decreasing in γ. This would suggest that a social planner that has a higher liking for capital would choose accumulation such that steady state consumption is lower. However, higher γ also implies more capital in the steady state and that effect is captured by the following arguments.

First notice that $\dot{k} = 0$ implies that steady state consumption must also satisfy

$$c^* = f(k) - \delta k. \tag{11}$$

From this, i.e. equation (10) and (11) we can determine the steady state capital stock k^* as satisfying

$$(f(k) - f'(k) \cdot k + \rho \cdot k)) \cdot \left(\frac{1}{1 + \gamma} \right) = f(k) - \delta k$$

where, of course, $w = f(k) - f'(k) \cdot k$. For our Cobb-Douglas economy we know that $f(k) = k^\alpha$, and $f'(k) = \alpha k^{\alpha - 1}$. Thus,

$$(1 - \alpha)k^{\alpha - 1} + \rho = (1 + \gamma)k^{\alpha - 1} - \delta(1 + \gamma).$$

Then it is not difficult to verify that the steady state capital stock is given by

$$k^* = \left(\frac{\alpha + \gamma}{(\gamma + 1)\delta + \rho} \right)^{\frac{1}{1 - \alpha}}. \tag{12}$$

and is increasing in γ. Thus, placing more welfare on capital holdings leads to a higher capital stock in the steady state. This does not seem very surprising. From this it follows that in an economy where there is a higher γ, the real return to capital in the steady state will be lower.

As "love for wealth" affects c^* through a direct and an indirect channel (through its effect on k^*) it is interesting to know what the condition is so that more capital raises overall steady state consumption. Now $c^* = f(k^*) - \delta k^*$ must hold in the steady state. Consumption c^* changes with γ according to

$$\frac{dc^*}{d\gamma} = f'(k^*) \cdot \frac{\partial k^*}{\partial \gamma} - \delta \frac{\partial k^*}{\partial \gamma}.$$

We know that $\frac{\partial k^*}{\partial \gamma} > 0$. Thus, the reaction of c^* depends on

$$\frac{dc^*}{d\gamma} \gtreqless 0 \quad \text{iff} \quad f'(k^*) \gtreqless \delta.$$

Lemma 2 *If $f'(k^*) = \delta$, then steady state consumption would be maximized and the "Golden Rule" of capital accumulation would be satisfied.*

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But from lemma 1 we know that the transversality condition requires $f'(k^*) > \delta$. Thus, this boils down to

$$f'(k) = \alpha k^{*\alpha-1} > \delta \quad \text{that is,} \quad \frac{\alpha[(\gamma+1)\delta+\rho]}{\alpha+\gamma} > \delta.$$

Simplification then reveals that γ has to satisfy

$$\widehat{\gamma} \equiv \frac{\alpha}{1-\alpha} \cdot \frac{\rho}{\delta} > \gamma. \tag{13}$$

Thus, if this condition is satisfied consumption and the capital stock in the steady state are higher than in an economy that does not feature love of wealth, i.e. where $\gamma = 0$. Up to $\widehat{\gamma}$ an increase in the love of wealth is accompanied by higher consumption and more physical capital in the steady state. Notice that the transversality condition requires that γ is smaller than $\widehat{\gamma}$. Interestingly, if γ equalled $\widehat{\gamma}$, then steady state consumption would be maximized with respect to γ. One easily verifies that if $\gamma = \widehat{\gamma}$, consumption would be maximal and the *Golden Rule* would hold. In this simple model we would then get the textbook condition that the the marginal product of capital equals the depreciation rate.

In this context it is well known that the welfare discount factor (the time preference rate ρ) leads one to choose a (steady state) consumption level that is smaller than the maximal one, i.e. the one that is associated with the Golden Rule. The reason is that ρ also captures impatience and so it is optimal in a Ramsey model to consume more today at the expense of higher consumption in the future. That implies a lower level of steady state consumption. In this model it is found that love of wealth is a counteracting force. In the limit when the social planner would have a love of wealth that is extremely close to $\widehat{\gamma}$, the social optimum would be to pursue an accumulation path that is *almost* equal to the one satisfying the Golden Rule.[15]

This holds, even if the the planner is more impatient. In this model more impatience (higher ρ) can be compensated in terms of steady state consumption by a higher γ as long as $\widehat{\gamma} > \gamma$ is satisfied.

Of course, higher consumption and more capital in the steady sate can only be accomplished if the investment share is higher. Under the condition $\widehat{\gamma} > \gamma$, we find

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[15]If γ is very close to $\widehat{\gamma}$ one may then be unable to distinguish empirically between a simple Solow model with an exogenous savings rate that followed the Golden Rule, and a Ramsey model with endogenous savings decisions featuring time preference and love of wealth.

that the consumption share in steady state is

$$\frac{c^*}{y^*} = \frac{f(k^*) - \delta k^*}{f(k^*)} = 1 - \delta k^{*1-\alpha} \quad \text{where} \quad y = f(k^*) = k^{*\alpha},$$

which will be lower for an economy with a higher γ. Consequently, the long-run investment share is higher in an economy where there is love of wealth.

Proposition 1 *More "love of wealth" (higher γ) increases the steady state capital stock k^* and implies a relatively lower steady state return to capital. It raises steady state consumption c^* if γ is not too large, that is, when $\gamma < \frac{\alpha}{1-\alpha} \cdot \frac{\rho}{\delta} \equiv \widehat{\gamma}$. If γ is very close to, but still less than $\widehat{\gamma}$, the accumulation path will almost obey the Golden Rule. However, if the love of wealth is too large ($\gamma > \widehat{\gamma}$), then an increase in γ would lower steady state consumption c^* in comparison to the maximum one, but then no optimum exists. The social planner always chooses a dynamically efficient accumulation path in the optimum. Furthermore, the optimal plan implies that an economy with more love of wealth will have a lower consumption and a higher investment share in the steady state.*

The proposition is interesting for the following reasons. The social planner's optimal policy is incompatible with an excessive love of wealth in the long run. A 'right' level of the love of wealth will lead to higher steady state consumption than when there is no love of wealth. If γ were too large, then a social planner would overaccumulate at the expense of consumption. This is not viable as an equilibrium.[16] A benevolent planner with the 'right' level of the love of wealth would not choose overaccumulation because she or he sees the potential for growth and higher consumption. By this reasoning the 'right' love of wealth is eventually beneficial for people and even though it implies a lower consumption share in the steady state. Clearly, these features of the model are in principle empirically testable. However, as the beneficial effect materializes only in a new steady state when γ is raised from $\gamma = 0$, one has to analyze what happens in the transition.

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[16]In the most extreme case when $\gamma \to \infty$, enslavement would ensue because the people would eventually die while producing more and consuming less, for example, by building ever more pyramids.

5 Transitional dynamics

For $\gamma \in [0, \frac{\alpha\rho}{(1-\alpha)\delta})$ the transitional dynamics of the economy can be described by the following two-dimensional system.

$$\frac{\dot{k}}{k} = \frac{f(k)}{k} - \frac{c}{k} - \delta \quad \text{and} \quad \frac{\dot{c}}{c} = f'(k) - (\rho + \delta) + \gamma\frac{c}{k}$$

plus the initial condition $k_0 > 0$ and the requirement that the transversality condition is met. For our Cobb-Douglas production case with $y = f(k) = k^\alpha$ and $y' = f'(k) = \alpha k^{\alpha-1}$ we then get for the equations of motion

$$\frac{\dot{k}}{k} = k^{\alpha-1} - \frac{c}{k} - \delta \quad \text{and} \quad \frac{\dot{c}}{c} = \alpha k^{\alpha-1} - (\rho + \delta) + \gamma\frac{c}{k}.$$

We will now log-linearize this system of equations around the steady state. Expressing the last expression in (natural) logs yields

$$\frac{d\ln k}{dt} = e^{-(1-\alpha)\ln k} - e^{\ln(c/k)} - \delta \tag{14}$$

$$\frac{d\ln c}{dt} = \alpha e^{-(1-\alpha)\ln k} + \gamma e^{\ln(c/k)} - (\rho + \delta). \tag{15}$$

In steady state we have $\frac{d\ln k}{dt} = \frac{d\ln c}{dt} = 0$ so that

$$e^{-(1-\alpha)\ln k} - e^{\ln(c/k)} = \delta \tag{16}$$

$$\alpha e^{-(1-\alpha)\ln k} + \gamma e^{\ln(c/k)} = \rho + \delta \tag{17}$$

have to hold. Solving for the steady state values yields

$$e^{\ln(c/k)} = \frac{\rho + (1-\alpha)\delta}{\alpha + \gamma} \tag{18}$$

$$e^{-(1-\alpha)\ln k} = \frac{\delta(1+\gamma) + \rho}{\alpha + \gamma}. \tag{19}$$

Now we linearize (14) and (15) to get

$$\begin{pmatrix} \frac{d\ln k}{dt} \\ \frac{d\ln c}{dt} \end{pmatrix} = \Delta \cdot \begin{pmatrix} d\ln k \\ d\ln c \end{pmatrix}$$

where $d \ln k = \ln k - \ln k^* = \ln(k/k^*)$ and $d \ln c = \ln c - \ln c^* = \ln(c/c^*)$ and

$$\Delta \equiv \begin{pmatrix} \Delta_{11} & \Delta_{12} \\ \Delta_{21} & \Delta_{22} \end{pmatrix} \equiv \begin{pmatrix} -(1-\alpha)e^{-(1-\alpha)\ln k} + e^{\ln(c/k)} & -e^{\ln(c/k)} \\ -\alpha(1-\alpha)e^{-(1-\alpha)\ln k} - \gamma e^{\ln(c/k)} & \gamma e^{\ln(c/k)} \end{pmatrix}_{c^*, k^*}$$

denotes the Jacobian of the system when in equilibrium. Evaluation at the steady state yields after substitution of equations (18) and (19), rearrangement and simplification that[17]

$$
\begin{aligned}
\Delta_{11} &= \frac{\alpha\rho - (1-\alpha)\gamma\delta}{\alpha+\gamma} \\
\Delta_{12} &= -\frac{\rho + (1-\alpha)\delta}{\alpha+\gamma} \\
\Delta_{21} &= -\frac{1}{\alpha+\gamma} \cdot (\alpha(1-\alpha)[\delta(1+\gamma) + \rho] + \gamma[\rho + (1-\alpha)\delta]) \\
\Delta_{22} &= \gamma \left(\frac{\rho + (1-\alpha)\delta}{\alpha+\gamma} \right)
\end{aligned}
\tag{20}
$$

For the (local) stability analysis we can determine the characteristic equation which is given by

$$\begin{vmatrix} \Delta_{11} - z & \Delta_{12} \\ \Delta_{21} & \Delta_{22} - z \end{vmatrix} = z^2 - z(\Delta_{11} + \Delta_{22}) + \Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21} = 0.$$

Notice that $tr\Delta = \Delta_{11} + \Delta_{22}$, which corresponds to the trace of the matrix Δ, and $|\Delta| = \Delta_{11}\Delta_{22} - \Delta_{12}\Delta_{21}$, which equals the determinant of Δ. The roots of the characteristic equation then satisfy

$$z_1, z_2 = \frac{tr\Delta \pm \sqrt{(tr\Delta)^2 - 4|\Delta|}}{2} \tag{21}$$

where is it well-known that

$$z_1 + z_2 = tr\Delta \quad \text{and} \quad z_1 z_2 = |\Delta|.$$

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[17]Δ_{ij} was verified, and the determinant in (22) as well as the roots in (23) were calculated in Mathematica 5.0. The source code is available in *low-1-math.nb* at *https://sites.google.com/site/rehmeguenther/resources/simulations-and-math-derivations/low2011*.

Thus, the sum of the roots equals the trace of Δ and the product of the roots equals its determinant. In our case it is not difficult to, but cumbersome to derive that[18]

$$tr\Delta = \rho \quad \text{and} \quad |\Delta| = -(a+\gamma)^{-1}\left((1-\alpha)((1-\alpha)\delta+\rho)(\delta(1+\gamma)+\rho)\right). \quad (22)$$

This means that the product of the roots is negative. Hence, one root is positive and one is negative. Thus, the solution to the characteristic equation requires

$$2z = \rho \pm \left[\rho^2 + 4(a+\gamma)^{-1}\left((1-\alpha)((1-\alpha)\delta+\rho)(\delta(1+\gamma)+\rho)\right)\right]^{1/2} \quad (23)$$

where z_1, the root that takes the positive sign, is positive, and z_2, the root that takes the negative sign, is negative.

The log-linearized solution for $\ln k$ then boils down to

$$\ln k = \ln k^* + \zeta_1 \cdot e^{z_1 t} + \zeta_2 \cdot e^{z_2 t}$$

where ζ_1 and ζ_2 are arbitrary constants of integration. As $z_1 > 0$, we need that $\zeta_1 = 0$ for $\ln k$ to approach $\ln k^*$ asymptotically. Then the other constant ζ_2, the one that is associated with $z_2 < 0$, can be obtained from the initial condition $\zeta_2 = \ln k_0 - \ln k^*$.[19] Thus, the evolution of k is described by

$$\ln k = \ln k^* + [\ln k_0 - \ln k^*] \cdot e^{z_2 t}, \quad \text{where} \quad z_2 < 0.$$

As $\ln y = \alpha \ln k$, one easily verifies that as a consequence output evolves according to

$$\ln y = \ln y^* + [\ln y_0 - \ln y^*] \cdot e^{z_2 t}, \quad \text{where} \quad z_2 < 0.$$

It is well-known that the negative root z_2 measures the *speed of convergence*. The latter tells one how fast the economy reaches its steady state. It is given by

$$2z_2 = \rho - \left[\rho^2 + 4(a+\gamma)^{-1}\left((1-\alpha)((1-\alpha)\delta+\rho)(\delta(1+\gamma)+\rho)\right)\right]^{1/2}.$$

---

[18]The details can be found at *sites.google.com/site/rehmeguenther/resources/simulations-and-math-derivations/low2011*.

[19]Here the assumption is that the initial values are close to the steady state. Although log-linear approximations are widely used in macroeconomics, the requirement that they apply only as approximations in the neighborhood of the steady sate can be regarded a disadvantage. See, for example, Barro and Sala–i–Martin (2004), p. 111.
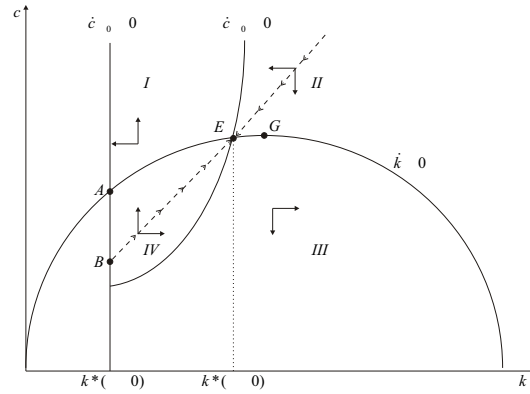
If z_2 is a large negative number then convergence takes place quickly. The opposite holds for small negative values of z_2. In appendix B it is shown that z_2 is increasing in γ. Thus, a higher γ implies a less negative z_2.

Proposition 2 *An increase in the love of wealth is accompanied by a decrease in the speed of convergence, that is, $\frac{dz_2}{d\gamma} > 0$. Thus, an economy with love of wealth takes longer to attain its steady state than an economy with no love of wealth.*

The proposition is interesting because in a normal Cobb-Douglas economy with logarithmic preferences and no love of wealth and a capital share of one third it is usually found that the convergence speed is too fast. See, for example, Barro and Sala–i–Martin (2004), ch. 2. One way out of this problem has been to argue that capital should be broadly defined. With a higher value of α for broad capital, it can be shown that the speed of convergence derived from a growth model with consumer optimization can match the empirically observed convergence speeds. In that context, the present setup provides an alternative route by arguing that a larger love of wealth would lead to a slower speed of convergence.

In order to see what the transitional dynamics of this system involve contemplate the following. Suppose we compare an economy that features no love of wealth with one that does. In particular, assume that once an economy without love of wealth has attained its steady state, the social planner will permanently change its preferences and permanently loves wealth from the position onwards where there was no love of wealth. Figure 1 displays the qualitative features of the dynamics if there is such a preferences shift. (The details of the phase diagram are derived in the appendix.)

Figure 1: The Pase Diagram

Without love of wealth the steady state position of the economy will be at point A with $\gamma = 0$ and the corresponding steady state capital stock $k^*(\gamma = 0)$. If the social planner changes his/her preferences and now features a (permanent) positive love of wealth, $\gamma > 0$, consumption will jump down to point B where it hits the saddle path associated with the dynamics of the economy with $\gamma > 0$. The saddle path is given by the dotted line with arrows. Thus, in order to attain the new steady state position at point E with (relatively) higher steady state consumption and capital stock $k^*(\gamma > 0)$, the consumption share will jump to a lower level. On the saddle path consumption and physical capital will rise over time. Thus, the economy will quickly get richer in comparison to the former steady state at $k^*(\gamma = 0)$. Furthermore, after the initial jump down in consumption, the latter will increase with the increase in capital. Eventually, that is, after some time in the transition period, there will be a a higher consumption level than the one associated the steady state level of consumption at point A. After that period consumption and capital will increase and be higher than in the steady state position for the economy without love of wealth. As has been analyzed above, the

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economy will end up in a new steady state that has higher capital, consumption and income than in the economy without love of wealth.

6 Decentralization

The setup of the model is such that a convex economy is analyzed. The utility function and the technology are (quasi)concave functions of its arguments. One can then invoke the Second Welfare Theorem and argue that the planner solution can be decentralized as a competitive general equilibrium of a private ownership economy. See Debreu (1959), Mas-Colell, Whinston, and Green (1995), ch. 16, and, more related to our growth context, Acemoglu (2009), ch. 5. Thus, the social planner solution can be realized as a market economy. This would only require that the agents have the same preferences as the social planner and where the social planner would have represented the agents' welfare in a benevolent way. Thus, all the results derived from the planner's solution would also carry over to the decentralized economy.

For the decentralized economy one can then interpret the more general term "love of wealth" as representing the "spirit of capitalism" as done in, for example, Zou (1994).[20] Thus, for a competitive market economy we may then conclude that the 'right' level of the "spirit of capitalism" is good in terms of long-run income and consumption. But it may take a little longer to realize these effects in comparison to an economy with less of that spirit. Furthermore, distinct ('right') "spirits of capitalism" may yield quite different economic outcomes in the long-run.

However, one main insight that the model can capture is that an *excessive* "spirit of capitalism" is not optimal, in the sense that it cannot really be realized as a long-run optimum in a dynamic economy.

7 Conclusion

In a simple optimal growth framework with Cobb-Douglas production technology and logarithmic utility which includes wealth as an argument it is analyzed how 'love of wealth' bears on optimal paths in a Solovian growth setup from a social planner's perspective. The following findings of the paper are noteworthy.

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[20]Notice that Zou contemplates a decentralized economy and also sometimes uses a single, constant value to represent the "spirit of capitalism".

First, for excessive 'love of wealth' no optimum exists for the social planner. If the intensity of the 'love of wealth' is 'right', i.e. below some critical threshold, there will be an equilibrium with higher per capita income, consumption and capital in comparison to the standard Ramsey-Cass-Koopmans world. In this case 'love of wealth' will definitely be beneficial for the agents and the social planner. In particular, long-run per capita consumption will be higher.

Second, with 'love of wealth' it is in principle possible to get arbitrarily close to the Golden Rule level of long-run consumption. The optimum implies that a dynamically efficient path will be followed and no overaccumulation will take place. There is a tradeoff between the rate of time preference (impatience) and the model's indicator of the 'love of wealth'. More impatience implies a lower and a higher intensity of the 'love of wealth' a higher steady state capital stock.

Third, an economy in which 'love of wealth' features in the preferences will have a slower speed of convergence. Thus, in comparison to the standard model it will take such an economy longer to attain its steady state. In a transition from a standard Ramsey-Cass-Koopmans economy to one that starts to like wealth from some point in time onwards and maintains this liking, it will take some time to realize the eventual benefits of having higher consumption and wealth. So initially some agents will not benefit, but from some time onwards they will.

Fourth, the social planner optimum can be decentralized as a private ownership, competitive economy. Building on previous work one may then identify 'love of wealth' with the "spirit of capitalism". All the results would then carry over for the decentralized economy. An important implication of the formal model then is that, as sometimes argued in recent policy debates on the virtues and vices of capitalism, an excessive "spirit of capitalism" precludes an optimum. In turn, a 'right' spirit will be quite beneficial for the long run.

The present paper should really be viewed as an additional move in the direction of focussing more on the role of preferences in accumulation processes. Of course, the analysis faces several caveats. The setup of the model is simple. Alternative utility and production functions might imply far more complicated equilibria or the lack thereof. 'Love of wealth' was captured by a constant. This begs the question how changes over time in the 'love of wealth' may bear on the optimal paths. These and other extensions of the model are left for further research.

A   Proof of the transversality condition

First, it is shown that $\lim\limits_{t \to \infty} e^{- \int_0^t (f'(k_v) - \delta) dv} = 0$ requires $f'(k_\infty) > \delta$ where $k_\infty = k^*$ and k^* denotes the finite steady state capital stock.

Suppose the economy starts off with some initial capital stock $k_0 < k^*$. At some point in time, called v^*, we will have $k_0 < k_{v^*} < k^*$. By assumption $f'(k)$ is a smooth and decreasing function of k. If $f'(k^*) > \delta$, then it must be that $f'(k_v) > \delta$ and $f'(k_0) > \delta$. Thus, $- \int_0^t (f'(k_v) - \delta) dv$ will be a negative number. Taking the limit one easily verifies that $\lim\limits_{t \to \infty} e^{- \int_0^t (f'(k_v) - \delta) dv} = 0$ if $f'(k^*) > \delta$ and $k_0 < k_{v^*} < k^*$.

Suppose the economy starts off with some initial capital stock $k_0 > k^*$ so that $k_0 > k_{v^*} > k^*$. Then it is true that

$$- \int_0^t (f'(k_v) - \delta) dv = - \int_0^{v^*} (f'(k_v) - \delta) dv - \int_{v^*}^t (f'(k_v) - \delta) dv \tag{24}$$

Starting from a large k_0, we have $f'(k_0) < \delta$. Thus, the expression in first integral on the right hand side will be a positive, but finite number, as the upper limit of integration is finite and $f'(k)$ is a monotone function. Thus,

$$- \int_0^{v^*} (f'(k_v) - \delta) dv = c_0$$

where c_0 is a positive constant which is independent of t. Given that $f'(k)$ is a decreasing function of k and as we consider k approaching k^* from $k_0 > k^*$ there will be some time v^* with $k_0 > k_{v^*} > k^*$ where $f'(k_{v^*}) > \delta$.

Now the second integral on the right hand side is equivalent to

$$- \int_{v^*}^t (f'(k_v) - \delta) dv = -t \left(\frac{\int_{v^*}^t (f'(k_v)) dv}{t} \right) + \delta(t - v^*).$$

Taking the limit yields

$$- \lim_{t \to \infty} t \cdot \lim_{t \to \infty} \left(\frac{\int_{v^*}^t (f'(k_v)) dv}{t} \right) + \lim_{t \to \infty} \delta(t - v^*).$$

Using l'Hopital's Rule one then finds

$$- \lim_{t \to \infty} t \cdot f'(k_\infty) + \lim_{t \to \infty} \delta(t - v^*) = - \lim_{t \to \infty} (f'(k_\infty) - \delta)t - \delta v^* = -\infty$$

because $f'(k_\infty) > \delta$ and $k^* = k_\infty$. Hence, if $f'(k^*) > \delta$, then $\lim\limits_{t \to \infty} e^{- \int_0^t (f'(k_v) - \delta) dv} = 0$.

Next, I want to show the convergence of

$$\lim_{t \to \infty} \int_0^t \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds. \tag{25}$$

In order to do that we will use a comparison test for convergence as, for instance, presented in Sydsaeter and Hammond (2002), p. 338, Theorem 9.7.1.[21] To this end assume first that

$$0 < k_0 < k_v < k^*$$

where the steady state capital stock k^* equals k_∞. Thus, we assume that there is some k_v at time v that is larger than the initial capital stock, k_0, but less than the steady state capital stock, $k^* = k_\infty$. The properties of the model and the functions are such that k will indeed (continuously) grow starting from the initial capital stock until reaching k^*. Thus, we know that the model implies

$$\lim_{v \to \infty} k_v = k^*$$

and that the transversality condition requires

$$f'(k^*) > \delta$$

which is true in the steady state. Furthermore, notice that $f'(k_v)$ is monotonically decreasing in k_v. Thus, there will be some time v^* out of $v \in [0, \infty)$ where

$$f'(k_{v^*}) < \delta + \rho \tag{26}$$

and we have that $f'(k_{v^*}) \geq f'(k_\infty) = f'(k^*)$ as $0 < k_0 < k_v < k^*$. Given this we can write (25) as

$$\lim_{t \to \infty} \int_0^{v^*} \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds + \lim_{t \to \infty} \int_{v^*}^t \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v) + \delta) dv} ds.$$

---

[21]The theorem is the following: Suppose that f and g are continuous for all $x \geq a$ and

$$|f(x)| \leq g(x) \qquad \text{for all } (x \geq a)$$

If $\int_a^\infty g(x) dx$ converges, then $\int_a^\infty f(x) dx$ converges, and

$$\left| \int_a^\infty f(x) dx \right| \leq \int_a^\infty g(x) dx.$$

The first integral is independent of t and finite as the limits of integration are finite and all the functions in side the integral are continuous. Thus

$$\lim_{t\to\infty} \int_0^{v^*} \left(\frac{\gamma}{k_s}\right) e^{-\int_0^s (\rho - f'(k_v) + \delta)dv} ds = c_1 \quad \text{where } c_1 \text{ is a finite constant.}$$

Thus, one has to check the second limit expression. This can be decomposed as follows

$$\lim_{t\to\infty} \int_{v^*}^t \left(\frac{\gamma}{k_s}\right) e^{-\int_0^{v^*} (\rho - f'(k_v) + \delta)dv - \int_{v^*}^s (\rho - f'(k_v) + \delta)dv} ds.$$

Now we note that $e^{-\int_0^{v^*} (\rho - f'(k_v) + \delta)dv}$ is independent of s and so t. Furthermore, it is finite and equal to some constant c_2. It can, thus, be pulled out of the limit expression, which yields

$$c_2 \cdot \lim_{t\to\infty} \int_{v^*}^t \left(\frac{\gamma}{k_s}\right) e^{-\int_{v^*}^s (\rho - f'(k_v) + \delta)dv} ds.$$

Thus, one only has to check the following integral for convergence.

$$\lim_{t\to\infty} \int_{v^*}^t \left(\frac{\gamma}{k_s}\right) e^{-\int_{v^*}^s (\rho - f'(k_v) + \delta)dv} ds = \lim_{t\to\infty} \int_{v^*}^t \left(\frac{\gamma}{k_s}\right) e^{\int_{v^*}^s (f'(k_v) - \rho - \delta)dv} ds.$$

In order to do that we note that for $v^* < v < s$ we have $f'(k_v) \le f'(k_{v^*})$. Furthermore,

$$\int_{v^*}^s (f'(k_v) - \rho - \delta)dv \le \int_{v^*}^s (f'(k_{v^*}) - \rho - \delta)dv = (f'(k_{v^*}) - \rho - \delta)(s - v^*)$$

where $(f'(k_{v^*}) - \rho - \delta) < 0$ by equation (26). As the exponential function e is a monotonically increasing function we have

$$e^{-\int_{v^*}^s (-f'(k_v) + \rho + \delta)dv} \le e^{-(-f'(k_{v^*}) + \rho + \delta)(s - v^*)}.$$

because for $v^* < v < s$ we have $f'(k_v) \le f'(k_{v^*})$. This and the fact that

$$f'(k_{v^*}) - \rho - \delta < 0$$

implies that the integral features absolute convergence. See, for example, Sydsaeter and Hammond (2002), p. 338, Theorem 9.7.1. For this one needs that

$$\int_{v^*}^\infty \left|\frac{\gamma}{k_s}\right| e^{-\int_{v^*}^s (-f'(k_v) + \rho + \delta)dv} ds \le \int_{v^*}^\infty \left|\frac{\gamma}{k_0}\right| e^{-(-f'(k_{v^*}) + \rho + \delta)(s - v^*)} ds \qquad (27)$$

The integral on the right hand side is given by

$$\left| \frac{\gamma}{k_0} \right| \cdot e^{(-f'(k_{v*})+\rho+\delta)v^*} \cdot \int_{v^*}^{\infty} e^{-(-f'(k_{v*})+\rho+\delta)s} ds, \tag{28}$$

where the expression in front of the integral is a constant. The integral, in turn, is of the type

$$\int_{v^*}^{\infty} e^{-as} ds, \quad \text{where} \quad a = (\rho + \delta - f'(k_{v*})) > 0. \tag{29}$$

Then it is not difficult to verify that

$$
\begin{aligned}
\int_{v^*}^{\infty} e^{-as} ds &= \lim_{t \to \infty} \int_{v^*}^{t} e^{-as} ds = \lim_{t \to \infty} \left. \frac{e^{-as}}{-a} \right|_{v^*}^{t} \\
&= \lim_{t \to \infty} \frac{1}{-a} \left(e^{-at} - e^{-av^*} \right) = \frac{e^{-av^*}}{a} < \infty.
\end{aligned}
$$

Thus, the integral on the right hand side of equation (27) converges. But by the comparison test for convergence it must then be that the integral on the left hand side of equation (27) converges, which in turn implies that

$$\lim_{t \to \infty} e^{-\int_0^t (f'(k_v)-\delta)dv} \cdot \lim_{t \to \infty} \int_0^t \left(\frac{\gamma}{k_s} \right) e^{-\int_0^s (\rho - f'(k_v)+\delta)dv} ds = 0. \tag{30}$$

This follows from the arguments above and in the text.

Finally it remains to check if the integral (25) converges if we approach the steady state from the right, that is, from

$$\infty > k_0 > k_v > k^*.$$

It is not difficult to see that then $-\int_0^s (\rho - f'(k_v)+\delta)dv$ will be a negative number in that case. Going through essentially the same steps as before it is not difficult to see that

$$\int_{v^*}^{\infty} \left| \frac{\gamma}{k_s} \right| e^{-\int_{v^*}^{s} (-f'(k_v)+\rho+\delta)dv} ds \leq \int_{v^*}^{\infty} \left| \frac{\gamma}{k^*} \right| e^{-(-f'(k_{v*})+\rho+\delta)(s-v^*)} ds \tag{31}$$

is needed for convergence. The integral on the right hand side of the inequality is convergent. Hence that (25) also converges if we start from $k_0 > k^*$

Together with the arguments in the main text, it then follows that the transversality condition is indeed met if $f'(k^*) > \delta$ and $f'(k^*) < \delta + \rho$ which is true in the steady state.

B Proof that the convergence speed is decreasing in γ

The negative root is given by

$$2z_2 = \rho - \left[\rho^2 + 4(a + \gamma)^{-1}\left((1 - \alpha)((1 - \alpha)\delta + \rho)(\delta(1 + \gamma) + \rho)\right)\right]^{1/2}.$$

This equation can be rearranged as

$$\rho^2 + 4(\alpha + \gamma)^{-1}\left(\Delta(\delta(1 + \gamma) + \rho)\right) = (\rho - 2z_2)^2 \quad \text{where} \quad \Delta \equiv ((1 - \alpha)((1 - \alpha)\delta + \rho).$$

Taking the differential with respect to z_2 and γ yields

$$\left(-4(\alpha + \gamma)^{-2}\Delta(\delta(1 + \gamma) + \rho) + 4(\alpha + \gamma)^{-1}\delta\Delta\right)d\gamma = (-2)(\rho - 2z_2)dz_2$$
$$4(\alpha + \gamma)^{-1}\Delta\left[-(\alpha + \gamma)^{-1}(\delta(1 + \gamma) + \rho) + \delta\right]d\gamma = (-2)(\rho - 2z_2)dz_2.$$

The expression in brackets is negative, because negativity requires

$$\delta(\alpha + \gamma) < \delta(1 + \gamma) + \gamma$$
$$\delta\alpha < \delta + \gamma$$

which is true as $\delta\alpha < \delta$. For the right hand side notice that, as $z_2 < 0$, the expression $(\rho - 2z_2)$ is positive. thus, it follows that z_2 is increasing in γ, i.e. $dz_2/d\gamma > 0$. Thus, a higher γ implies a lower speed of convergence.

C The phase diagram

The qualitative features of the dynamic system are characterized as follows. The system of differential equations is

$$\frac{\dot{k}}{k} = k^{\alpha-1} - \frac{c}{k} - \delta \quad \text{and} \quad \frac{\dot{c}}{c} = \alpha k^{\alpha-1} - (\rho + \delta) + \gamma\frac{c}{k}.$$

When $\dot{k} = 0$ in the steady state we get $c = k^\alpha - \delta k$. Taking the differential with respect to k and c yields

$$dc = (\alpha k^{\alpha-1} - \delta)dk.$$

As the expression in the round brackets is positive for low k and negative for sufficiently large k, the $(\dot{k} = 0)$-line will be inverted U-shaped as depicted in figure 1. This is a standard result.

When $\dot{c} = 0$ we get $\gamma c = (\rho + \delta)k - \alpha k^\alpha$. Taking the differential for this expression yields

$$\gamma \cdot dc = \left((\rho + \delta) - \alpha^2 k^{\alpha-1}\right) dk. \tag{32}$$

This means that $\frac{dc}{dk}_{|\dot{c}=0} < 0$ for very low k and $\frac{dc}{dk}_{|\dot{c}=0} > 0$ for sufficiently large k. For the arguments in the text initial k for the preference shift corresponds to the one where k^* is not a function of γ. The latter is given by

$$k^*_{|\gamma=0} = \left(\frac{\alpha}{\delta + \rho}\right)^{\frac{1}{1-\alpha}}. \tag{33}$$

But then it is easy to verify that $\frac{dc}{dk}_{|\dot{c}=0} > 0$ holds for any $k > k^*_{|\gamma=0}$. Starting from $k^*_{|\gamma=0}$ it also turns out that $\frac{d^2c}{dk^2}_{|\dot{c}=0} > 0$ so that the $(\dot{c} = 0)$-line is a convex function of k as depicted in figure 1.

Turning to the dynamic adjustment notice that

$$\frac{d\dot{k}}{dc} = -1. \tag{34}$$

Thus, if c is slightly above the $(\dot{k} = 0)$-line, k will fall. That explains the arrows pointing westward above that line. Similarly, if c is slightly below the $(\dot{k} = 0)$-line, k will rise, which explains the arrows pointing eastward above the $(\dot{k} = 0)$-line.

As concerns the $(\dot{c} = 0)$-line notice that

$$d\dot{c} = \left[\alpha(\alpha - 1)k^{\alpha-2} - \gamma \frac{c}{k^2}\right] c \cdot dk.$$

The expression in square brackets is negative. Thus, $\frac{d\dot{c}}{dk}_{|\dot{c}=0} < 0$. So if k is to the right of the $(\dot{c} = 0)$-line, consumption decreases. That explains the downward pointing arrows on the right of the $(\dot{c} = 0)$-line. Furthermore, if k is to the left of the $(\dot{c} = 0)$-line, consumption increases. That explains the upward pointing arrows on the left of the $(\dot{c} = 0)$-line.

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