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Assessing the short and medium term response of monetary shocks in Argentina

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ABSTRACT

Monetary shocks generated by a monetary authority financing fiscal deficits cause a number of impacts on various economic variables, particularly on exchange and inflation rate. And while inflation is not an aspect which constitutes a serious concern in the world, it really is in Argentina, primarily from 2006 when the country returned to experiencing high rates of price changes. This was what led us to find the determinants of inflation in the long run (see Descalzi and Neder, 2015). Assessing the assumptions related to a cash-in-advance model for a small open economy with seigniorage and following McCandless (2008), we found a long run relationship between inflation, money issuing, nominal exchange rate and fiscal deficit, meaning that inflation, nominal exchange rate and the government imbalances are driven by the same trend. In this paper, being more interested in short run, we deepen the understanding on the inflationary process in Argentina during the period Q1 2004- Q2 2015 by adjusting the impulse response functions to evaluate the time along which the inflation rate attains its steady-state level after a monetary shock occurs. We use quarterly data.

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I. Introduction

The Cash-in-advance models (henceforth CIA) and the influence of inflation tax were introduced by Cooley and Hansen (1989). This model requires the previous existence of money for consumption held by people. As is pointed out by McCandless (2008) what describe a CIA is a two member's family: one of them is the shopper and the other is the producer. The shopper purchases goods at the market and the producer, using capital and labor, elaborate goods that are sold to other families. This is the way in which money arrives to family and will be used next time to purchases. However, in this paper, we use an extended CIA model, since we introduce a real government budget constraint to get the long term relationship between inflation, nominal exchange rate and fiscal deficit, considering the use of money. Other papers, like Hodrick, R. et al (1991) and Carmichael, B (1989) make predictions about the stochastic properties of endogenous variables such as the velocity of circulation of money, the rate of inflation, and real and nominal interest rates, meanwhile Calvo (1987) analyzes the balance of payment crisis in a CIA model.

In a former paper (Descalzi and Neder, 2015) it was showed that monetary shocks generated by the monetary authority financing fiscal deficits cause a number of impacts on various economic variables, particularly on exchange and inflation rate. And while inflation is not an aspect which constitutes a serious concern in the world, it really is in Argentina, primarily from 2006 when the country returned to experiencing high rates of price changes. This was what led us to find the determinants of inflation in the long run. In this paper, being more interested in the short run, we deepen the understanding on the inflationary process in Argentina during the period Q1 2004- Q2 2015 by adjusting the impulse response functions to evaluate the time along which the inflation rate attains its steady-state level after a monetary shock occurs. In this sense, we worked with a model that includes funding not only from issuing money, but also (domestic and foreign) public debt. So, in the nominal budget constraint for Families, these two variables will be considered, and the nominal exchange rate as well.

The paper is organized as follows. In the next section we present the theoretical approach for the CIA model. In section III we present the empirical approach, making considerations about the estimation of cointegrating vectors, elasticities and the structural decomposition. In section IV we show the results and in sections V and VI the concluding remarks and bibliography.

II. Theoretical approach

The Cash-in-Advance constraint for Families is expressed in nominal terms as follows:

$$P_t c_t = M_{t-1} \quad (1)$$

being P_t the general price level, c_t the real consumption level, and M_{t-1} the nominal stock of money previously accumulated, needed to buy goods and services in this period.

Additionally, the binding nominal budget constraint for Families, i.e. their real and financial wealth applied to get new one, is:

$$P_t \omega_t = P_t [f(k_{t-1}) + (1 - \delta)k_{t-1}] + (1 + i_{t-1})B_{t-1} + (1 + i_{t-1}^e)B_{t-1}^{eP} tcn_t - T_t = P_t k_t + M_t + B_t + B_t^{eP} tcn_t \quad (2)$$

Real wealth is represented by ω_t . The real wealth includes the production (generated by inputs accumulated in the previous period), and the net (of depreciation) capital level, while the financial wealth considers Net Domestic (B_{t-1}) and External (B_{t-1}^{eP}) Assets plus interests. Since External Assets are valued in foreign money, they are multiplied by the nominal exchange rate.

Taking equation (2) in real terms:

$$\omega_t = [f(k_{t-1}) + (1 - \delta)k_{t-1}] + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{1+i_{t-1}^e}{1+\pi_t} b_{t-1}^{eP} tcn_t - \tau_t = k_t + m_t + b_t + b_t^{eP} tcn_t \quad (2')$$

Considering constant returns to scale:

$$f(k_{t-1}) = \lambda_{t-1} k_{t-1}^\theta h_{t-1}^{1-\theta} = w_{t-1} h_{t-1} + r_{t-1} k_{t-1} \quad (3)$$

Inserting (3) in (2')

$$\omega_t = [w_{t-1} h_{t-1} + r_{t-1} k_{t-1} + (1 - \delta)k_{t-1}] + \frac{1+i_{t-1}}{1+\pi_t} b_{t-1} + \frac{1+i_{t-1}^e}{1+\pi_t} b_{t-1}^{eP} tcn_t - \tau_t = k_t + m_t + b_t + b_t^{eP} tcn_t \quad (2'')$$

From (2'') we can get h_{t-1} as follows:

$$h_{t-1} = \frac{[k_t - (1+r_{t-1}-\delta)k_{t-1}] + m_t + [b_t - \frac{1+i_{t-1}}{1+\pi_t} b_{t-1}] + tcn_t [b_t^{eP} - \frac{1+i_{t-1}^e}{1+\pi_t} b_{t-1}^{eP}] + \tau_t}{w_{t-1}} \quad (4)$$

In order to maximize the following particular utility function

$$u(c_t, 1 - h_t) = \ln c_t + B h_t \quad \text{being } B < 0 \quad (5)$$

Subject to (1) and (4), we get the Bellman equation

$$V(k_{t-1}, m_{t-1}, b_{t-1}, b_{t-1}^{eP}, \lambda_t, g_t) = \max_{k_t, m_t, b_t, b_t^{eP}} \left[\ln \left(\frac{m_{t-1}}{1+\pi_t} \right) + B \left(\frac{[k_t - (1+r_{t-1}-\delta)k_{t-1}] + m_t + [b_t - \frac{1+i_t-1}{1+\pi_t}b_{t-1}] + \Delta tcn_t [b_t^{eP} - \frac{1+i_t^e-1}{1+\pi_t}b_{t-1}^{eP}] + \tau}{w_{t-1}} \right) \right] + \beta V(k_t, m_t, b_t, b_t^{eP}, \lambda_{t+1}, g_{t+1}) \quad (6)$$

The first order conditions obtained are the following:

$$\frac{1}{\beta} = 1 + r_t - \delta \quad (7)$$

$$m_t = -\frac{\beta}{B} \bar{w} \quad (8)$$

$$\frac{1}{\beta} = \frac{1+i_t}{1+\pi_{t+1}} \quad (9)$$

$$\frac{1}{\beta} = (1 + \Delta tcn_{t+1}) \frac{1+i_t^e}{1+\pi_{t+1}} \quad (10)$$

From (7), (9), and (10) it is clear that in the optimum the returns of three assets (k, b and b^{eP}) are equal, through an arbitrage behavior followed by Families in the open economy: $1 + r_t - \delta = \frac{1+i_t}{1+\pi_{t+1}} = (1 + \Delta tcn_{t+1}) \frac{1+i_t^e}{1+\pi_{t+1}}$. We find that in the short run the gross investment return (net of the depreciation rate) is equal to the domestic interest rate discounted by the inflation rate corresponding to next period. The rationale of the cash-in-advance model is that the current consumption is covered with the real balances that the individual holds at the beginning of the period (supplied by the fiscal deficit). Then, after having earned the interest rate, the representative agent has to "wait" one period (in which is affected by the rate π_{t+1}) to build new (real) cash balances to be consumed in $t+1$. In the same way, the (gross) rate of return of capital (net of depreciation rate) is equal to the gross international rate multiplied by the (gross) depreciation rate in $t+1$ and also discounted by gross inflation rate in $t+1$. In a cash-in-advance model the individual had to consider "in advance" the forthcoming interest rate and the depreciation rate, as well.

From (8), the demand of real balances by Families is proportional to their steady state salaries, as it is usually found in the literature.

Once the optimal behavior rule of the Families is determined, in order to know the effects of fiscal shocks on inflation, the budget constraint (that relates fiscal expenditure with money issuing) is added to the analysis. To get the long term relationship between inflation, nominal exchange rate and fiscal deficit, the first order conditions are replaced in the real government budget restriction.

$$(g_t - \tau_t) = \left[1 - \frac{1}{\varphi^B} (1 + i_{t-1})\right] b_t + \left[1 - \frac{1}{\varphi^{B^{eg}}} (1 + i_{t-1}^e) tcn_t\right] b_t^{eg} + \left[1 - \frac{1}{\varphi^M}\right] m_t$$

$$(g_t - \tau_t) = \left[1 - \frac{1}{\varphi^B} (1 + i_{t-1})\right] b_t + \left[1 - \frac{1}{\varphi^{B^{eg}}} (1 + i_{t-1}^e) tcn_t\right] b_t^{eg} - \left[1 - \frac{1}{\varphi^M}\right] \left[\frac{\beta}{B} \bar{w}\right] \quad (11)$$

where φ^j , with $j = B, B^{eg}$ represents the gross growth rate of the B and B^{eg} stocks.

In the long run, under the assumption of transversality condition for Families behavior³, the gross growth rate of the mentioned stocks could have an upper bound given by the gross interest rate, domestic and external (this last one multiplied by the nominal exchange rate), respectively. If this upper bound is met, the case resembles that of a close economy, where we are in a near pure seigniorage.

Doing a short run analysis, when the gross growth rate of the stocks is below the bound, the gross growth rate of money issue increases. Funding needs through money increase since the government decides not to use all the possibility of funding with bonds. Hence the importance of reducing the fiscal deficit. It could even be a situation in which the interest rate rise temporarily (in which case the country would not get more debt at that level because of such temporary situation). However, in that moment, should support upward pressure on prices because it will increase the monetary issuing, and it appears rather contradictory that as the interest rate increases, borrowing capacity expand.⁴

Now, in order to do an analysis of long-term, the current values of the variables in equation (11) are replaced by their steady state values. This will determine the borrowing limit that has the economy in the long term, which is given by the transversality condition that determines that at most the growth rate of the stock of debt is equal to the gross average interest rate on long term.

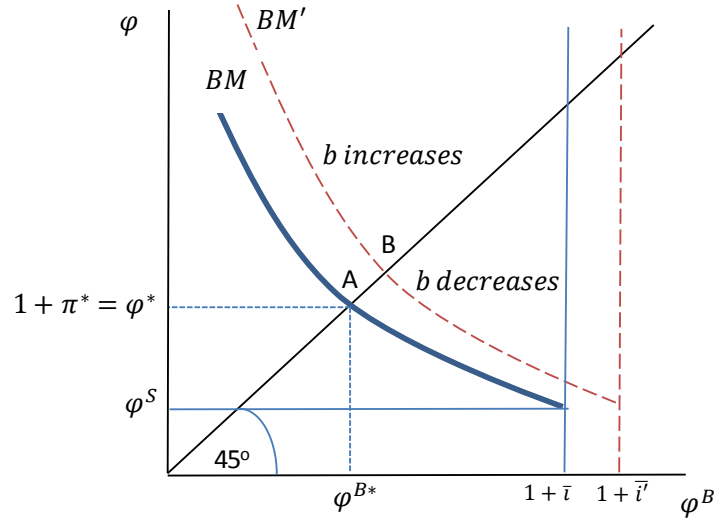
Under the assumption that the economy does not fully utilize the availability of funding, the gross interest rate is higher than the growth rate of nominal debt stock (i.e., the brackets of the first and second term are negative).

The long-term equilibrium found can be represented by the following diagram. In the vertical axis we represent the rate of money growth (which in turn is equal to the gross rate of inflation) and in the abscissa the growth rate of nominal bond stock. The inverse

³ $\lim_{T \rightarrow \infty} \frac{B_{t+T+1}^j}{(1+i_t^j) \dots (1+i_T^j)} = 0$, with $j = \text{domestic and external public debt}$.

⁴ When the international interest rate increases, the discount value of the issued debt is reduced. It means that the financing possibilities increases due to a kind of wealth effect (the liabilities are reduced).

relationship between the gross growth rate of money and the gross growth rate of the stock of bonds is the curve BM, which represents the trade off in funding.



The higher the growth rate of bonds, the lower the rate of monetary emission. In the limit, i.e. when the rate of growth of nominal bond stock is approaching its maximum, that it means, is equal to the nominal interest rate, we are in the presence of pure seigniorage case presented by McCandless (2008). To achieve the steady state value of the fiscal deficit be constant, i.e., avoiding explosive paths, the stocks of debt in real terms should be also constant. This situation would be compatible with long-term budget objective. This is graphically identified by a line of 45°. Thus, values of growth rate of bonds equal to the rate of money growth are determined. In the point where this curve intersects the BM curve (point A), the long-term equilibrium is determined for which public debt levels in real terms are constant and the fiscal deficit assumes no explosive behavior. Therefore, it is verified that:

$$(1 + i) > \varphi^B = 1 + \pi = \varphi \quad (12)$$

being i the average long-term interest rate, φ^B the growth rate of debt, π the long-run inflation rate, and φ the growth rate of money.

If an increase in the international interest rate occurs (for example, due to a shock), the line reflecting the limit for the nominal stock of debt should move to the right. In this case, using less external funding, and due to the raise in the cost of internal public funding, it will be necessary more money issuing to maintain the level of fiscal deficit

funding. This will move the BM curve to the right getting the point B. In this point, the growth rates of bonds, money and inflation all increase at the same proportion.

In this sense, it is expected that the greater the fiscal deficit, the greater the inflation rate, and the model also enables us to show that greater levels of fiscal deficits tend to appreciate the local currency. This is similar to the situation explained for an increase in the international interest rate.

III. Empirical approach

The first step is to estimate the reduced form of Vector Correction Model (VECM). In this stage we estimate the cointegration rank, and finally the number of permanent shocks in the system. The cointegration rank (r) indicates the number of common trends, while the difference between the number of (integrated) variables (K) and the cointegration rank (let's say, $K-r$) indicates the long-run trends that pull the forces in the system. Additionally, the analysis indicates how the main variables are related across those trends: given that the cointegrating matrix is singular, it is necessary to impose identifying restrictions to estimate their coefficients. These restrictions are imposed according to the economic theory, indicating the causality relationships between them. It is also possible to infer (by testing the coefficients of the loading matrix) whether the relevant variables are endogenous in the estimated system.

In the second stage we perform an impulse-response analysis. The question we raise here is, once we determine the trends in the system, which are the structural variables that drive them? Sometimes, the structural variables are "hidden" behind those observed in the (reduced form of the) system -for example, in a Blanchard-&-Quah-type scheme, the transitory shocks are leaded by (non-observable) demand shocks, while the permanent shocks are entirely driven by (non-observable) supply shocks-. Given that in a VECM *all* the variables are related between themselves, it is necessary to impose identifying restrictions to link structural shocks to those observed in the reduced form VEC. Thus, even though the structural shocks match the observed ones, it is necessary to impose identifying restrictions (based on economic theory) to stress the structural relationships in the system, leaving aside other possibilities considered *a priori* in VEC the reduced-form. In the identifying stage, it is mandatory to take into account the results obtained at the time of adjusting the reduced-form of a VECM; that is, if two

transitory shocks were found for example, the identified matrix of long run response must display two zero-rows, to represent the null impact that the identified variables (which are supposed to be affected by these kind of shock) have in the long run.

Once the structural coefficients that measure the short and long run response of the variables have been identified, it is possible to evaluate how fast a structural shock to the variable we are interested in is absorbed by the system.

Given the structural VEC form:

$$\mathbf{A}\Delta\mathbf{Y}_t = \Pi^* \mathbf{Y}_{t-1} + \Gamma_1^* \Delta\mathbf{Y}_{t-1} + \Gamma_2^* \Delta\mathbf{Y}_{t-2} + \dots + \Gamma_p^* \Delta\mathbf{Y}_{t-p} + \mathbf{v}_t$$

where \mathbf{A} is an invertible matrix of order $(k \times k)$ that reflects the instantaneous relationships between the variables included in the vector of endogenous variables \mathbf{Y}_t , whose order is $(k \times 1)$. $\Delta\mathbf{Y}_t$ (and its lags) represents the vector of variations in endogenous variables, Π^* is a matrix that includes the cointegration relationships, Γ_i^* is a matrix of order $k \times k$ which represents impacts of transitory shocks. \mathbf{v}_t is the error vector of order k which is equal to $\mathbf{B}\boldsymbol{\varepsilon}_t$ being \mathbf{B} a matrix of order $(k \times k)$ and $\boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_{1t} \quad \boldsymbol{\varepsilon}_{2t} \quad \dots \quad \boldsymbol{\varepsilon}_{kt})'$ a vector of orthogonal structural shocks ($\boldsymbol{\varepsilon}_{it} \boldsymbol{\varepsilon}_{jt} = 0$ for $i \neq j$).

The reduced form is obtained premultiplying the previous equation by \mathbf{A}^{-1} :

$$\Delta\mathbf{Y}_t = \mathbf{A}^{-1}\Pi^* \mathbf{Y}_{t-1} + \mathbf{A}^{-1}\Gamma_1^* \Delta\mathbf{Y}_{t-1} + \mathbf{A}^{-1}\Gamma_2^* \Delta\mathbf{Y}_{t-2} + \dots + \mathbf{A}^{-1}\Gamma_p^* \Delta\mathbf{Y}_{t-p} + \mathbf{A}^{-1}\mathbf{v}_t$$

or:

$$\Delta\mathbf{Y}_t = \Pi \mathbf{Y}_{t-1} + \Gamma_1 \Delta\mathbf{Y}_{t-1} + \Gamma_2 \Delta\mathbf{Y}_{t-2} + \dots + \Gamma_p \Delta\mathbf{Y}_{t-p} + \boldsymbol{\mu}_t$$

where $\boldsymbol{\mu}_t = \mathbf{A}^{-1}\mathbf{B}\boldsymbol{\varepsilon}_t$. To get the estimation of the reduced form, first we impose

$2k^2 - \frac{k(k+1)}{2} = k^2 + \frac{k(k-1)}{2}$ restrictions on $\mathbf{A}\boldsymbol{\mu}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$ to identify the structural form.

Then, we invert the reduced form to obtain the vector of endogenous variables in terms of the structural errors.

III.1. Estimating the cointegrating vectors

In this section we pursue the objective of analyzing the joint dynamics between the real exchange rate, the inflation and the public expenditure both in the short and long term.

In a first stage the coefficients of the cointegrating vectors are computed. In a second stage, the impulse-response functions are shown to explain the dynamics of the inflation rate that follows a shock in fiscal deficit.

We assume that the government exploits entirely its financing opportunities by fulfilling the transversality condition (so the expressions between brackets in equation 11 -related to bonds- equal to zero). Thus, under this circumstance the seigniorage depends upon the real long term expenditure selected by the government (we exclude both the external and internal government debt). In this empirical approximation we control for government revenue (T) -given that in Argentina we do not represent a case of pure seigniorage- and also for the shocks to GDP (to take into accounts the productivity shocks that could affect the economy). In this paper we have selected the period Q1 2004- Q3 2014 by adjusting the impulse response functions to evaluate the time along which the inflation rate attains its steady-state level after a monetary shock occurs, and using quarterly data.

The vector of endogenous variables is given by $Y_{t-1} = (rer_{t-1}, \pi_{t-1}, T_{t-1}, g_{t-1}, gdp_{t-1})'$. rer_{t-1} is (the log of) growth real exchange rate; π_{t-1} is (the log of) the gross inflation rate, which was calculated as the variation in the consumer price index (CPI) elaborated by the Government of San Luis Province⁵. g_{t-1} is (the log of) public expenditure in real terms (at 2004 prices) with data from the Ministry of Economics.

The first objective we pursue here is to determine the cointegration rank r^* . This will give us also the number of permanent shocks that affects the system, given by $K-r^*$. In the following table the Johansen trace and the Lütkepohl and Saikkonen tests are driven to obtain the appropriate cointegration rank. It can be seen that the result of the test depends on the combination between the optimal lag order and the deterministic variables added to each test. All in all, a cointegration rank of order 3 seems to be a reasonable choice, although we also could have chosen $r=4$. Our hypothesis here is that the macroeconomic aggregates in Argentina are driven by the shocks to productivity (these probably are global shocks) and by (country-specific) demand driven by public expenditure.

⁵ This Government gained good reputation in calculating the CPI because it followed a similar methodology to that elaborated by the INDEC (the National Bureau of Statistics in Argentina) after this organism had the loss of credibility in 2007.

Table 1
Cointegration tests
 $Y_{t-1} = (rer_{t-1}, \pi_{t-1}, T_{t-1}, g_{t-1}, gdp_{t-1})'$

Variables	K	Deterministic	Lag*	r*	K-r* (permanent shocks)
rer, π, T, g, gdp	5	C, SEAS, T	5 (AIC, HQ, SC)	3 (90%; Joh) - 2 (90%; S&L)	2-3
rer, π, T, g, gdp	5	C, SEAS	5 (AIC, HQ, SC)	3 (99%; Joh) - 4 (90%; Joh) - 4 (90%; S&L)	2-1
rer, π, T, g, gdp	5	SEAS	6 (AIC, HQ, SC)	-----	-----
rer, π, T, g, gdp	5	C	6 (AIC, HQ, SC)	4 (99%; Joh) -4 (90%; S&L)	1-1

Note: Joh refers to Johansen Trace test. L&S refers to Lütkepohl and Saikkonen test.

III.1.a. The elasticities with respect to expenditure and gross domestic product

Having determined the cointegration rank we move to estimate the cointegration relations $\beta'Y_{t-1}$. It is mandatory to stress here that the estimation of the VECM form is only performed to check the overall consistency of the theoretical scheme and that of the estimation method, as well. The estimation of the cointegrating vector requires to set adding identifying restrictions given that the matrix $\Pi = \alpha'\beta$ is singular with rank=r*.

The following table shows the estimates of the of cointegration vectors. Under the assumption that rank (β) = 3, we estimate the cointegration equations $ec_{t-1} = \beta'Y_{t-1}$, so that:

$$rer_{t-1} = -\beta_{41}g_{t-1} - \beta_{51}gdp_{t-1}$$

$$\pi_{t-1} = -\beta_{42}g_{t-1} - \beta_{52}gdp_{t-1}$$

$$T_{t-1} = -\beta_{43}g_{t-1} - \beta_{53}gdp_{t-1}$$

It is expected that $-\beta_{41}$ be negative: if the (long run) public expenditure increases (causing a greater fiscal deficit financed by issuing new money), the (gross) rate of growth of *rer* diminishes. The coefficient $-\beta_{42}$ is thought to be positive because as *g* increases, the (long run) inflation rate tends to be higher. Finally, it is expected a positive relationship between government revenues and the public expenditure ($-\beta_{43} > 0$). The estimated coefficients are shown in table 2. All the variables are expressed in logarithms, then the coefficients are elasticities. Given that in the cointegration

equations all the variables are located in the RHS, for a right interpretation of the coefficients, the sign of the elasticities should be inverted.

Firstly, the estimated coefficient for the response of (the log of) the rate of change of *rer* to the public expenditure *g* is found to be negative ($-\beta_{41} < 0$) and significantly different from zero, as expected. Secondly, the corresponding of the inflation rate is positive and significantly different from zero (as it was also expected). This strengthens the role of the fiscal irresponsibility of government, which derives in a higher long run inflation rate, reinforcing the hypothesis of the existence of seigniorage. Finally, the long-term relationship between the public expenditure and the government revenue is positive as expected, being the elasticity close to one.

Table 2
Coefficients of cointegration relations $\beta'Y_{t-1}$
 $Y_{t-1} = (rer_{t-1}, \pi_{t-1}, T_{t-1}, G_{t-1}, PIB_{t-1})'$

Cointegration Equation	Coefficients of the cointegrating vector β :				
	rer_{t-1}	π_{t-1}	T_{t-1}	g_{t-1}	gdp_{t-1}
EC _{1,t-1}	1.000	(0.000)	(0.000)	0.526	-2.458
	(0.000)	{0.000}	{0.000}	(0.106)	(0.457)
	{0.000}	[0.000]	[0.000]	{0.000}	{0.000}
	[0.000]	(0.000)	(0.000)	[4.976]	[-5.378]
EC _{2,t-1}	0.000	1.000	0.000	-0.383	1.753
	(0.000)	(0.000)	(0.000)	(0.076)	(0.329)
	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
	[0.000]	[0.000]	[0.000]	[-5.030]	[5.332]
EC _{3,t-1}	0.000	0.000	1.000	-0.988	0.512
	(0.000)	(0.000)	(0.000)	(0.025)	(0.109)
	{0.000}	{0.000}	{0.000}	{0.000}	{0.000}
	[0.000]	[0.000]	[0.000]	[-39.330]	[4.716]

(Std. Dev.) {p - Value} [t - Value] *** p<0.01, ** p<0.05, * p<0.1.

The Akaike, Final Prediction Error, Hannan-Quinn and Schwarz Criteria indicated that the optimal VAR lag length is equal to 5. The cointegration test was run using Johansen (Trace) and Lütkepohl and Saikkonen (L&S) procedures. The null hypothesis $H_0: \text{rank}(\beta) = 3$ cannot be rejected, so that the VEC was specified assuming that the cointegration rank is equal to 3. Remaining VEC's specification details are as follows: deterministic variables: CONST S1 S2 S3, endogenous lags (in differences): 4, sample range: [[2005 Q2, 2014 Q3], T = 38, estimation procedure: Two stage. 1st=Johansen approach, 2nd=3SLS. Further estimation details are available upon request. Estimations were carried out using J-Multi.

Source: Own calculations.

The table also reports the estimates for the coefficients that measures the impact of the *gdp* on the (the log of) the rate of change of *rer*, on the rate of inflation π , and on fiscal revenue ($-\beta_{51}, -\beta_{51}, -\beta_{51}$). The response of the rate of change of the *rer* is positive, suggesting that a productivity gain will provoke in the long run an increase in the rate of

change in the *rer*. The response of π to *gdp* in the long run is negative and significantly different from zero. This finding is consistent with sign of the previous coefficient: as productivity increases, the inflation rate lowers, increasing the real exchange rate. Finally, the coefficient that measures the relation between *gdp* and fiscal revenue is negative and significantly different from zero, indicating that the (permanent) shocks to productivity would be associated with tax reductions.

III.1.b. Structural decomposition

In this section we perform a structural decomposition of the Vector Error Correction (VEC) model to obtain the response of *rer*, π and T to a structural shock in the public expenditure. The main objective is to analyze how these variables react in the short run to account for the number of periods needed to achieve the steady state.

The vector of endogenous variables can be decomposed to take into account short and long effects of the shocks in the following way (Lütkepohl et al; 2005):

$$\mathbf{Y}_t = \Xi \sum_{i=1}^t \boldsymbol{\mu}_{t-i} + \Xi^*(L)\boldsymbol{\mu}_t + \mathbf{Y}_0^*$$

To identify the shock, we need to impose some restrictions on the matrix that reflects the long-run responses, given by $\Xi\mathbf{B}$ (see the methodological Appendix for more details). Given that

$$\boldsymbol{\varepsilon}_t = [\varepsilon_t^{rer} \quad \varepsilon_t^{\pi} \quad \varepsilon_t^T \quad \varepsilon_t^G \quad \varepsilon_t^{gdp}]'$$

In the work we suggest the following form:

$$\Xi\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & 0 \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix}$$

We have imposed 16 zero-restrictions on this matrix. Asterisks * indicates freely estimated parameters. This shape indicates that the shocks to *rer*, π and T do not have permanent impact in the long-run. Only shocks to public expenditure (ε_t^G) and to product (ε_t^{gdp}) would have permanent effects. This could be interpreted in the following

way. The number of the cointegration relationships given the number of transitory shocks in the system; then, the number of permanent shocks is given by the total number of variables ($K=5$) minus the cointegration rank ($r=3$). The rank of $\Xi \mathbf{B}$ should be equal to 2, which implies entering three zero-column vectors. Additionally, it was assumed that ε_t^{gdp} does not affect the inflation rate in the long run, to complete the number of restrictions needed for this matrix.

The zero-restrictions on the matrix \mathbf{B} (that measures the sign and magnitude shocks in the moment of the impact) are imposed as follows:

$$\mathbf{B} = \begin{bmatrix} * & 0 & 0 & * & * \\ * & * & 0 & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix}$$

where it is supposed that (according to the equation $\mu_t = \mathbf{B}\varepsilon_t$) at the moment of impact the shocks on π and T do not affect the *rer*. These assumptions would be in line with the economic process we are trying to measure. However, it would be necessary to analyze whether the sign of the freely estimated coefficients is in line with the theoretical background.

IV. Results

In this section results are shown. Firstly, the sign of significance of the estimated parameters of the matrices \mathbf{B} and $\Xi \mathbf{B}$ are analyzed. Secondly, the impulse-response functions are drawn to depict the speed of the response of *rer*, π and T to the shocks to public expenditure. The following table shows how the variables respond "at the moment of impact" to shocks to g . The rate of change of the *rer* responds positively to a shock to g . It would be expected a negative sign, because the inflation tends to appreciate the domestic currency. The inflation and the government revenue also respond positively to a shock to g , as expected.

Table 3
Estimated \mathbf{B} matrix.
Structural VAR estimation results.

<i>rer</i>	π	T	<i>g</i>	<i>gdp</i>
0.0098	0.0000	0.0000	0.0153	-0.0098
[4.2533]	[0.0000]	[0.0000]	[1.3554]	[-3.1314]
-0.0066	0.0005	0.0000	0.0132	0.0087
[-3.0659]	[1.6492]	[0.0000]	[1.3502]	[2.7007]
-0.0303	0.0201	0.0005	0.0183	0.0000
[-3.9315]	[1.7553]	[0.3359]	[1.2378]	[0.0017]
0.0093	0.0142	-0.0100	0.0084	0.0111
[1.7941]	[1.9897]	[-1.2781]	[1.9160]	[1.8562]
-0.0012	-0.0024	0.0038	-0.0065	0.0086
[-1.1105]	[-1.9484]	[1.3581]	[-1.3384]	[3.6577]

(Std. Dev.) [bootstrap t - Value]

This is a B-model with long run restrictions Long run restrictions provide(s) 9 independent restriction(s). Contemporaneous restrictions provide(s) 6 additional restriction(s). ML Estimation, Scoring Algorithm (see Amisano & Giannini (1992)). Convergence achieved after 14 iterations. Log Likelihood: 1240.4333 Structural VAR is just identified
Source: Own calculations.

The Table 4 reports the accumulated (long-run) effects associated to shocks to *g* and *gdp*. The response of the (the rate of change of) the real exchange rate is negative; while the corresponding response of the rate of inflation is positive as expected. The reported bootstrap t-values are not so high. However, we are interested in analyzing the evolution of the variables in the short run, given that the accuracy of the forecasted responses lowers as time goes by.

Table 4
Estimated $\Xi \mathbf{B}$ matrix.
Structural VAR estimation results.

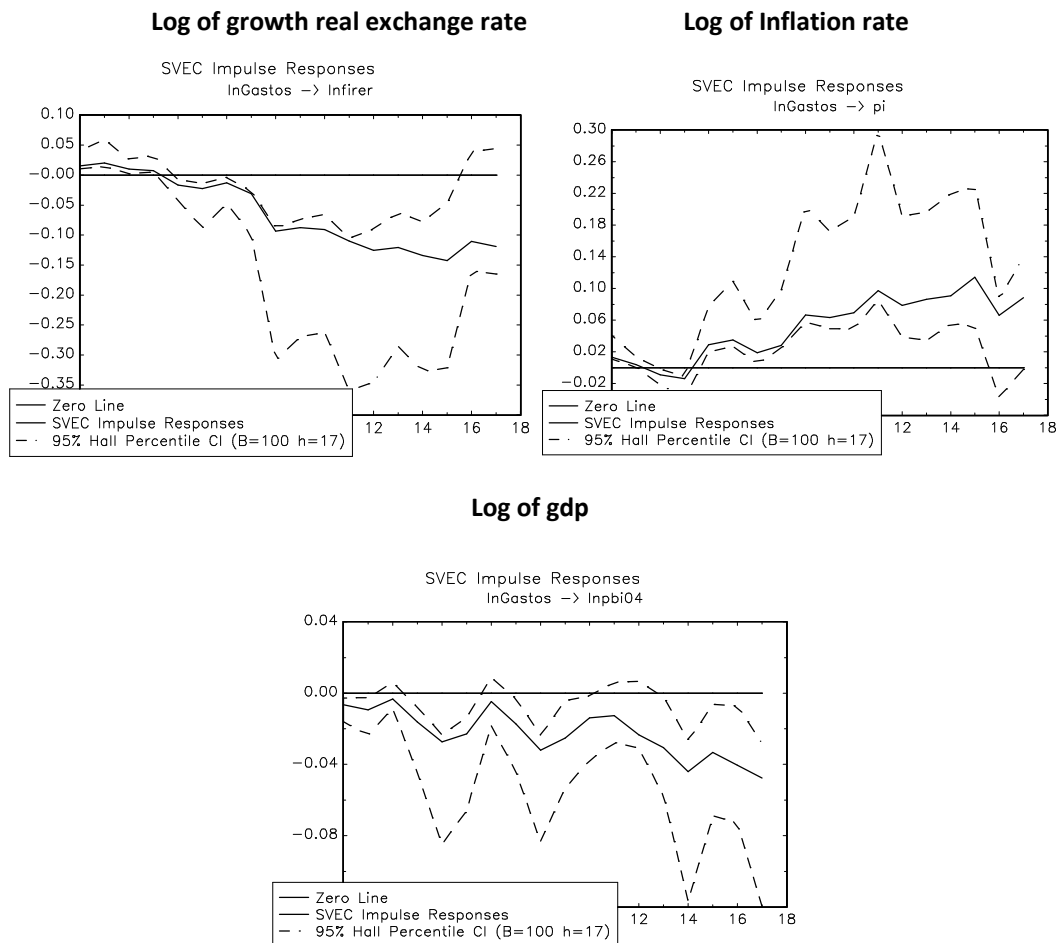
<i>rer</i>	π	T	<i>g</i>	<i>gdp</i>
0.0000	0.0000	0.0000	-0.1230	0.0003
[0.0000]	[0.0000]	[0.0000]	[-0.1281]	[3.1978]
0.0000	0.0000	0.0000	0.0866	0.0000
[0.0000]	[0.0000]	[0.0000]	[0.1281]	[0.0000]
0.0000	0.0000	0.0000	-0.1073	0.0246
[0.0000]	[0.0000]	[0.0000]	[-0.1319]	[3.1978]
0.0000	0.0000	0.0000	-0.1513	0.0281
[0.0000]	[0.0000]	[0.0000]	[-0.1313]	[3.1978]
0.0000	0.0000	0.0000	-0.0824	0.0061
[0.0000]	[0.0000]	[0.0000]	[-0.1294]	[3.1978]

[bootstrapped t - Value]

This is a B-model with long run restrictions Long run restrictions provide(s) 9 independent restriction(s). Contemporaneous restrictions provide(s) 6 additional restriction(s). Structural VAR estimation results. ML Estimation, Scoring Algorithm (see Amisano & Giannini (1992)). Convergence after 14 iterations. Log Likelihood: 1240.4333 Structural VAR is just identified
Source: Own calculations.

The Figure 1 shows short-run dynamics of the growth of the real exchange rate, the inflation rate and the *gdp* following a shock to the public expenditure. It can be seen that the growth of the real exchange rate tends to decrease after experiencing a hike at the moment of impact. Thus, the steady-state growth rate of the exchange rate would be lower if the government increases its long-run level of expenditure. The (long-run) inflation rate tends to increase as a consequence of a fiscal boom, suggesting that the greater the level of the public expenditure is in the long run, the higher the inflation rate needed to finance the increase in *g*. Finally, it can be seen that the *gdp* respond negatively when a shock to *g* occurs.

Figure 1
Response of the *rer*, the inflation and the *gdp* to a shock in the fiscal deficit



Source: Own calculations. Indef equals to *def*. $\pi=\pi$. All the confidence intervals were obtained by bootstrap, according to the method of Hall, see Lütkepohl (2004) for further details. The regressions we run using software J-Multi. The impulse-response function was calculated on the basis of a reduced form VECM calculated with 1 Lag for the period 2002-2011, using quarterly data.

V. Concluding Remarks

In this paper we tackle the issue of seigniorage in a small open economy. Our theoretical framework relies on a cash-in-advance model with seigniorage, in which the government is allowed to issue both internal and external debt at the international interest rate. Our main purpose was to analyze the role of the exchange rate and the interest rate at the time of choosing a financing scheme. The derived first order conditions show a new version of the uncovered interest rate parity. We conclude that if the government wants to increase the expenditure it will have to exploit all of its financing opportunities by issuing debt according to the bound suggested by the transversality condition in order to avoid a higher steady-state inflation rate. In this model, however, we do not set any rule that Central Bank could use to lead the exchange-rate policy. So, the theoretical issues we present here would allow different exchange rate regimes. In the empirical application, and extending a previous work, we analyzed the response of the exchange rate and the inflation to changes in the public expenditure g . We found that the inflation rate reacts positively to a shock in the public expenditure; while in the steady state the growth of the real exchange rate tends to diminish to a lower steady state level.

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METHODOLOGICAL APPENDIX

The corresponding moving average (MA) form is given by (Beveridge-Nelson, Granger Decomposition Theorem, page 251):

$$\mathbf{Y}_t = \Xi \sum_{i=1}^t \boldsymbol{\mu}_{t-i} + \Xi^*(L) \boldsymbol{\mu}_t + \mathbf{Y}_0^*$$

where $\Xi = \beta_{\perp} (\alpha'_{\perp} (I_K - \sum_{i=1}^{p-1} \Gamma_i) \beta_{\perp})^{-1} \alpha'_{\perp}$ (where the symbol \perp indicates orthogonal complement)

is a $(K \times K)$ matrix that represents the long run effects of forecast error of impulse responses.

$\Xi^*(L) \boldsymbol{\mu}_t = \sum_{j=0}^{\infty} \Xi_j^* L^j \boldsymbol{\mu}_t = \sum_{j=0}^{\infty} \Xi_j^* \boldsymbol{\mu}_{t-j}$ is an $I(0)$ process. The parameter matrices Ξ_j^* are

determined by the model parameters (see Lütkepohl et al. (2005) for more details). \mathbf{Y}_0^*

contains the initial values. In this case we have that $\text{rank}(\Xi) = K - r = 5 - 3 = 2$. This means

that there are three ($r=3$) shocks with transitory effects and only two shocks with permanent effects in the system. To obtain the impulse-response functions replace in the moving

average form $\boldsymbol{\mu}_t = \mathbf{A}^{-1} \mathbf{B} \boldsymbol{\varepsilon}_t$; assuming that $\mathbf{A} = I_5$, it is still necessary to impose $K(K-1)/2=10$

identifying restrictions (Lütkepohl et al (2005); page 168). There are $r(K-r)=6$ independent

restrictions to be imposed on the matrix that reflects the long-run responses $\Xi \mathbf{B}$. It is

necessary to impose $(K-r)[(K-r)-1]/2=1$ further restrictions on this matrix. Finally, the

remaining $r(r-1)/2=3$ correspond to be imposed on the matrix \mathbf{B} (that shapes the responses of endogenous variables at moment of impact of structural shocks). The identifying conditions

are imposed taking into account the economic reasoning involving the pass-through process.

Given that,

$$\boldsymbol{\varepsilon}_t = [\varepsilon_t^{rer} \quad \varepsilon_t^{\pi} \quad \varepsilon_t^T \quad \varepsilon_t^G \quad \varepsilon_t^{pib}]'$$

the matrix that measures the long run impact is given by:

$$\Xi \mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & * & * \end{bmatrix}$$

Given that there are only two shocks with long-run impacts, the first four columns represent four zero-column vectors (these are the 6 independent restrictions). These restrictions imply that only the shocks to expenditure and to *piib* have effects on the long-run. Finally, the additional restriction on this matrix indicates that shocks to *piib* have not long-run impact on *rer* in the long-run. The asterisks indicate freely estimated parameters.

The remaining 3 restrictions on the matrix **B** are imposed as follows:

$$\mathbf{B} = \begin{bmatrix} * & 0 & 0 & * & * \\ * & * & 0 & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix}$$

where it is supposed that (according to the equation $\mu_t = \mathbf{B}\varepsilon_t$) in the moment of impact: first, the shocks on π and T do not affect the *rer*; second, the shocks on T do not affect π ; These assumptions would be in line with the economic process we are trying to measure. However, it would be necessary to analyzed whether the sign of the freely estimated coefficients agree with the theoretical background.